5.4 What is the most entropic ideal-gas binary mixture?

For a binary mixture we have $n_A + n_B = n$, $x_A = n_A/n$, and $x_B = n_B/n = 1 - x_A$. Thus Eq. (5.5) translates into

$$\begin{aligned} \Delta_{mix} S_m^{id} =& \quad -R\left(x_A \ln x_A + x_B \ln x_B\right) = -R\left(x_A \ln x_A + (1-x_A)\ln(1-x_A)\right) \\ \text{max } \Rightarrow 0 = \frac{d}{dx_A}\Delta_{mix} S_m^{id} =& \quad -R\left(\ln x_A + 1 - \ln(1-x_A) - 1\right) = -R \ln\left(\frac{x_A}{1-x_A}\right) \end{aligned} \tag{5.8}$$

This is only fulfilled for $x_A = x_B = 0.5$ which is the only maximum. This result is quite intuitive because a 50% mixture allows for the most efficient disorder of particle arrangement. Since

$$\Delta_{mix} G_m^{id} = -T \Delta_{mix} S_m^{id} \tag{5.9}$$

the minimum of the Gibbs potential is found for $x_A = x_B = 0.5$ as well and it is more pronounced for higher temperature.