

3.25 Fundamental equations and exact differentials

We now relate Eq. (3.60) to the partial derivatives.

$$\begin{aligned}
 dU &= TdS - pdV = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV \Rightarrow \left(\frac{\partial U}{\partial S}\right)_V = T \quad \left(\frac{\partial U}{\partial V}\right)_S = -p \\
 dF &= -SdT - pdV = \left(\frac{\partial F}{\partial T}\right)_V dT + \left(\frac{\partial F}{\partial V}\right)_T dV \Rightarrow \left(\frac{\partial F}{\partial T}\right)_V = -S \quad \left(\frac{\partial F}{\partial V}\right)_T = -p \\
 dH &= TdS + Vdp = \left(\frac{\partial H}{\partial S}\right)_p dS + \left(\frac{\partial H}{\partial p}\right)_S dp \Rightarrow \left(\frac{\partial H}{\partial S}\right)_p = T \quad \left(\frac{\partial H}{\partial p}\right)_S = V \\
 dG &= -SdT + Vdp = \left(\frac{\partial G}{\partial T}\right)_p dT + \left(\frac{\partial G}{\partial p}\right)_T dp \Rightarrow \left(\frac{\partial G}{\partial T}\right)_p = -S \quad \left(\frac{\partial G}{\partial p}\right)_T = V
 \end{aligned} \tag{3.61}$$

These results clarify which exact differential with which restrictions have to be used/combined to get proper results related to the thermodynamic potentials.