

3.22 Calculation of the free energy of an ideal gas

We can calculate the free energy (Helmholtz energy) starting with the state function and the inner energy of an ideal gas

$$pV = NkT \quad , \quad (3.50)$$

$$U = 3/2NkT \quad . \quad (3.51)$$

(Note: In this notation U is not a potential, since T is not a coordinate of U !). Using Eq. (3.50) we get

$$p = -\frac{\partial F}{\partial V} = \frac{NkT}{V} \quad , \quad (3.52)$$

i.e.

$$F(V, N, T) = -NkT(\ln(V) + K(N, T)) \quad . \quad (3.53)$$

The function $K(N, T)$ must still be calculated. Combining Eq. (3.51),

$$U = F + TS, \text{ and } S = -dF/dT \quad (3.54)$$

we find

$$\begin{aligned} \frac{3}{2}NkT = U &= -NkT(\ln(V) + K(N, T)) - T \left[-Nk(\ln(V) + K(N, T)) - NkT \frac{\partial K(T, N)}{\partial T} \right] \\ &= NkTT \frac{\partial K(T, N)}{\partial T} \quad . \end{aligned} \quad (3.55)$$

Consequently

$$K(T, N) = 3/2 \ln(T) + K'(N) \quad , \quad (3.56)$$

leading to

$$F(V, N, T) = -NkT [\ln(V) + 3/2 \ln(T) + K'(N)] \quad . \quad (3.57)$$

Successively integrating the state functions of a system allows to calculate the thermodynamic potential.

This procedure is necessary because in contrast to an electrical potential there is no way of measuring a thermodynamic potential directly. We therefore have to measure all "forces" in each state, thus determining the state functions which allow us to calculate the potential.