2.10 $C_p - C_V$ for perfect gases

Here we will prove a relation for $C_p - C_V$ which holds for nearly all solids and liquids and for ideal gases; so we discuss a very general feature of heat capacities. Including the expansion coefficient α of Eq. (2.6) in Eq. (2.11) we get

$$dU = \pi_T dV + C_V dT \quad \Rightarrow \quad \left(\frac{\partial U}{\partial T}\right)_p = \pi_T \left(\frac{\partial V}{\partial T}\right)_p + C_V = \pi_T\, \alpha V + C_V \tag{2.27}$$

We learned already:

- α: Expansion coefficient is small for solids and liquids.
- π_T: Is small for gases and zero for perfect gases.

$$\Rightarrow \quad \left(\frac{\partial U}{\partial T}\right)_p \approx C_V \tag{2.28}$$

$$\text{Thus using} \quad \left(\frac{\partial U}{\partial T}\right)_p \approx \left(\frac{\partial U}{\partial T}\right)_V = C_V \quad \text{we get} \quad C_p - C_V = \left(\frac{\partial H}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_V \approx \left(\frac{\partial H}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_p \tag{2.29}$$

$$\text{i.e.} \quad C_p - C_V = \left(\frac{\partial (U + pV)}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial (U + n\,RT)}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_p = n\,R \tag{2.30}$$

So for many materials the specific heat capacities C_p/n and C_V/n just differ by a constant R.