Solution to Exercise 8.1-1

Exponential Growth

Illustration

1. The output of the solar cell industry in **2006 - 2008** grew by **40 %** per year. Let's assume that all solar cells installed in **2007** produced a total energy of **0.1 GW /year**. Calculate (and plot) the installed power as a function of time up to **2050** for growth rates of **α' = 20 %**, **30 %**, **40 %**, and **50 %**. What is the proper equation?

- The general equation is $P(t) = P_0 \cdot \exp(\alpha \cdot t)$ and we know $P(t = 0\ a) = 100$ MW and $P(t = 1a) = 100\ \text{MW} + (\alpha'/100) \cdot 100$ MW; α' is the given growth rate in %
- It follows that

$$P(t = 1\text{a}) = 100\ \text{MW} \cdot \exp(\alpha \cdot 1\text{a} \cdot \text{a}^{-1})$$

$$= 100\ \text{MW} + (\alpha'/100) \cdot 100\ \text{MW}$$

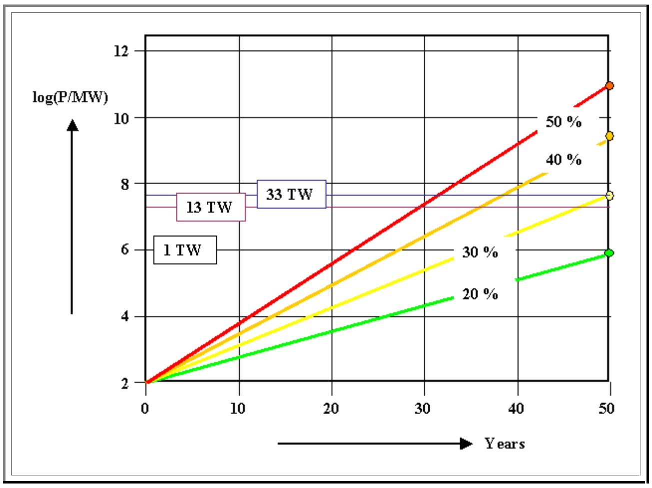
$$\alpha = \ln(1 + \alpha'/100)\ \text{a}^{-1}$$

$$= (0.182;\ 0.262;\ 0.336;\ 0.405)\ \text{a}^{-1}$$

for growth rates of 20% ; 30%; 40%; 50%

2. Calculate (and plot) the installed power as a function of time up to **2050** for growth rates of **20 %**, **30 %**, **40 %**, and **50 %**.

- That's easy and we do it, of course, in a **log** ***P*(*t*)** plot. What we get looks like this:



3. What follows form the results with respect to the world-wide power scenario as described in the link??

- It follows that with the present growth rate of **40 %** all of the world's energy demands can be produced by solar cells in **35 - 38** years - be it the present **13 TW** or the predicted **33 TW**
- That looks like a "Milchmädchenrechnung" (i.e. very naive), because that's what it is. If we can sustain a growth rate of **40 %** for **30 - 40** years remains to be seen. It's unlikely, but not impossible. The semiconductor industry, for example, sustained a growth rate of about **30 %** by now for more than **35** years, and no end is in sight.

4. Plot the demand for **Si**, assuming that a standard (**1000 x 1000 x 0.1**) **mm^3 Si** solar cell generates **10 W** on average. Will there be enough **Si**? How do the amounts of **Si** needed compare to other essential raw materials?

- The volume is **10^5 mm^3 = 100 cm^2**. With a density of **2.33 g/cm^3** we have **23.2 g/W**.
- The present (**2007**) production of (solar grade) **Si** per year is roughly **20.000 to = 2 · 10^{10} g**; corresponding to **862 MW**. If we want to produce **1 TW,** we would need **23.2 · 10^{13} g = 23.2 · 10^7 to** of **Si**.
- That looks like a lot of **Si**. Yes, but look at the present world production of:
    - **Iron / Steel:** ≈ **780 · 10^6 to**.
    - **Coal:** ≈ **5 000 · 10^6 to**.
    - **Al** ≈ **22 · 10^6 to**.
- So a few million tons of **Si** is definitely within present day capabilities