

Gain Coefficient

Advanced

The **gain coefficient** describes how the density of photons, $u_{\nu}(\mathbf{z})$, changes as they propagate along the \mathbf{z} -direction. The definition [implicitly used before](#) was

$$\frac{d u_{\nu}(\mathbf{z})}{d z} = g_{\nu} \cdot u_{\nu}(\mathbf{z})$$

The physical process for the change of the photon density was stimulated emission (increasing the density) and fundamental absorption (decreasing the density). Both effects we combined into a [net emission rate](#) which expresses the balance of emission or absorption rates taking place the photons propagate in z direction:

$$R_{se}^{net} = R_{se} - R_{fa} = R_{se}^{net}(\mathbf{z}) = R(\mathbf{z})$$

For the individual emission rates R_{se} and R_{fa} [we had simplified equations](#), however, not expressively as a function of \mathbf{z}

$$R_{fa} = A_{fa} \cdot N_{eff} \cdot u_{\nu} \cdot \Delta \nu \cdot [1 - f_{h \text{ in } \nu}(E^{\nu}, E_F^h, T)] \cdot [1 - f_{e \text{ in } c}(E^c, E_F^e, T)]$$

$$R_{se} = A_{se} \cdot N_{eff} \cdot u_{\nu} \cdot \Delta \nu \cdot [f_{e \text{ in } c}(E^c, E_F^e, T)] \cdot [f_{h \text{ in } \nu}(E^{\nu}, E_F^h, T)]$$

From a somewhat more detailed look at the inversion condition in [an advanced module](#), using e.g. the proper density of states instead of effective densities, we obtained "better" equations which we are now going to use:

$$R_{fa}(E^{\nu}, E^c) = \left(A_{fa} \right) \cdot \left(D_{\nu}(E^{\nu}) \cdot \Delta E^{\nu} \cdot [1 - f(E^{\nu}, E_F^h)] \right) \cdot \left(D_c(E^c) \cdot \Delta E^c \cdot [1 - f(E^c, E_F^e)] \right) \cdot \left(u(\nu) \right)$$

$$R_{se}(E^c, E^{\nu}) = \left(A_{se} \right) \cdot \left(D_{\nu}(E^{\nu}) \cdot \Delta E^{\nu} \cdot [1 - f(E^{\nu}, E_F^h)] \right) \cdot \left(D_c(E^c) \cdot \Delta E^c \cdot f(E^c, E_F^e) \right) \cdot \left(u(\nu) \right)$$

Summing up (= integration) for all possible transitions gives for R^{net}

$$R^{net} = A \cdot u_{\nu} \cdot \int_{E_c} [D_c(E^{\nu} + h\nu) \cdot D_{\nu}(E^{\nu}) \cdot [f(E^{\nu} + h\nu, E_F^e) + f(E^{\nu}, E_F^h) - 1] \cdot dE^{\nu}]$$

The **change** in the density of the photons is now directly given by

$$\frac{\partial u_{\nu}(\mathbf{z}, t)}{\partial t} = R^{net}$$

which we can write as

$$\frac{\partial u_{\nu}(\mathbf{z}, t)}{\partial t} = \frac{\partial u_{\nu}(\mathbf{z}, t)}{\partial z} \cdot \frac{\partial z}{\partial t} = R^{net}$$

We use the partial derivative signs ∂ to make clear that we have more than one variable.

This may look at bit strange. What does it mean?

- It means that the density of a bunch of photons that are contained in some volume element at some point \mathbf{z} is given by the product of the change in density along \mathbf{z} that they experience in their travel, times the rate with which they change their position and this means that

$$\frac{\partial \mathbf{z}}{\partial t} = v_g = \text{group velocity of the photons .}$$

Look at a simple analogy:

- When you and your friends travel as a group from Kiel to Munich, starting with some amount of money m_{Kiel} , which will certainly change by the time you reach Munich, you have a certain value of the money *gradient* dm/dl along the length l of your path.
- Your *rate of spending*, dm/dt , depends on how *much* you spent along the way ($= dm/dl$) times how *fast* you spent it ($= dl/dt$),

$$\frac{dm}{dt} = \frac{dm}{dl} \cdot \frac{dl}{dt}$$

- and dl/dt is just the velocity with which you move.

For $\{\partial u_{\nu}(\mathbf{z}, t)/\partial \mathbf{z}\} \cdot \{\partial \mathbf{z}/\partial t\}$ we already have the independent expression that defined the gain coefficient [from above](#), and we also have the lengthy expression for R^{net} . Inserting it yields

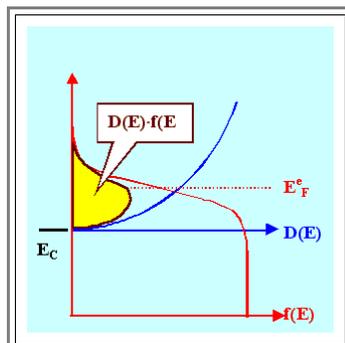
$$R^{\text{net}} = g_{\nu}(\mathbf{z}) \cdot v_g \cdot u_{\nu} = A \cdot u_{\nu} \int_{E_c} \left(D_C(E^{\nu} + h\nu) \cdot D_V(E^{\nu}) \right) \cdot \left(f(E^{\nu} + h\nu, E_F^e) + f(E^{\nu}, E_F^h) - 1 \right) \cdot dE^{\nu}$$

- from which we obtain the final formula

$$g_{\nu} = \frac{A}{v_g} \cdot \int_{E_c} \left(D_C(E + h\nu) \cdot D_V(E^{\nu}) \right) \cdot \left(f(E^{\nu} + h\nu, E_F^e) + f(E^{\nu}, E_F^h) - 1 \right) \cdot dE^{\nu}$$

This looks complicated (actually, it *is* complicated) - but it is a clear recipe for calculating g .

- Essentially, the integral as a function of the frequency ν scales with the density of electrons in the conduction band and the density of holes in the valence band exactly $h\nu$ electron volts below. Both values increase if the Quasi Fermi energies move deeper into the bands.
- The integral runs over the valence band, summing up all energy couples between the valence band and the conduction band that are separated by $h\nu$; it will thus be a function of ν . For some ν , depending on the carrier concentration, it will have a maximum. This is easy to see if we consider the distribution of electrons (or holes) in the conduction (or valence) band.



- In this example for the conduction band, the quasi Fermi energy is somewhere above the band edge. The product of the Fermi distribution with the density of states (here as the [standard parabola](#) from the free electron gas approximation) always will give a pronounced maximum somewhere between E_C and E_F^c . The same thing happens for the holes in the valence band.
- The energy difference between the two maxima will be the energy or frequency where g_V is largest. If we increase the carrier concentrations, i.e. if we move the quasi Fermi energies deeper into the bands, g_V will increase too, and the maximum value shifts to somewhat larger energies.

▸ All things considered, we now have:

- A good idea of how to calculate g_V and what we need to know for the task.
- A good idea of the general behavior of g_V and what we have to do in a qualitative way to change its value to what we want.
- A pretty good grasp why g_V looks the way [we have drawn it](#) - without justification - in a backbone module.