2.3.4 Useful Relations

There is no way to cover all relevant semiconductor physics within the scope of this course. This subchapter provides some important or useful relations needed for the understanding of the topics.

- It also serves as the "gate" to a number of modules providing additional information.

- This subchapter therefore is more open than the other ones; it will fill out and sprout a network in connection with the lecture course that cannot be predicted by now.

Einstein Relation

We have encountered the Einstein relation before. It is of such fundamental importance that we give *two* derivations: one in this paragraph, another one in an advanced module.

First, we consider the internal current (density) in a material with a *gradient* of the carrier density (n_e or n_h).

- Fick's first law then tells us that the diffusion-driven *particle* current $j_{p,diff}$ is given by

$$ j_{p,diff} = - D_{e,h} \cdot \nabla n_{e,h} $$

- If the particles are carrying a charge q, this *particle* current is also an **electrical current** (which obviously is a **diffusion** current, then), given by

$$ j_{e,h} = q \cdot j_{p,diff} = - q \cdot D_{e,h} \cdot \nabla n_{e,h} $$

- Considering only the one-dimensional case for electrons (i.e. $q = -e$; holes behave in exactly the same way with $q = +e$), we have

$$ j_e(x) = e \cdot D_e \cdot \frac{d n_e(x)}{dx} $$

Since there can be no *net* current in a piece of material just lying around (which nevertheless might still have a density gradient in the carrier density, e.g. due to a gradient in the doping density), the carriers displaced by diffusion always generate an electrical field that will drive the other carriers back.

- Any field $E(x)$ (written in mauve to avoid confusion with energies) now will cause a (so far one-dimensional) current given by

$$ j = \sigma \cdot E(x) = q \cdot n(x) \cdot \mu \cdot E(x) $$

- With σ = conductivity, μ = mobility.

- Note that the result is always the technical current density, which is positive for positive charge carriers. Yet this equation also works for electrons because for them, effectively, two minus signs cancel: one from their negative charge and the other from their direction of movement opposite to the electric field. This means that in a strict sense, their mobility should be negative. However, in this equation one only considers positive charges and positive mobilities – also for electrons. Therefore, to use ths equation in full generality, we write it as

$$ j = e \cdot n(x) \cdot \mu \cdot E(x) $$

The *total* (one-dimensional) current in full generality is then

$$ j_{total}(x) = e \cdot n(x) \cdot \mu \cdot E(x) - q \cdot D \cdot \frac{d n(x)}{dx} $$

- We will need this equation later.

For our case of *no net current* and only *fields caused by the diffusion current*, both currents have to be equal in magnitude:

$$e \cdot n(x) \cdot \mu \cdot E(x) \;=\; q \cdot D \cdot \frac{dn(x)}{dx}$$

- This is an equation that comes up repeatedly; we will encounter it again later when we derive the Debye length.
- Note that in this equation, the sign on the right-hand side depends on the type of charge carriers (since $q = \pm e$). This is balanced on the left-hand side by the direction of the electric field.

Now we are stuck. *We need some additional equation in order to find a correlation between D and μ.*

- This equation is the *Boltzmann distribution* (here used as an approximation to the *Fermi distribution*), because we have *equilibrium* in our material.
- However, we also know that, in this equilibrium situation, we have spatially varying charge carrier densities and electric fields. We know such a situation from the p–n junction in equilibrium. There, this was only possible due to the band bending, i.e. that the band edges were functions of the lateral position. This we also consider here.
- Just to derive the relation to the electric field in the above equation, for the moment we just consider the case of electrons as majority carriers. For their local density it holds that

$$n(x) \;=\; N_{\text{eff}} \cdot \exp - \frac{E_C(x) - E_F}{kT}$$

- Differentiation of the Boltzmann distribution gives us

$$\frac{dn}{dx} \;=\; - N_{\text{eff}} \cdot 1/(kT) \cdot \frac{dE_C(x)}{dx} \cdot \exp - \frac{E_C(x) - E_F}{kT}$$

$$\frac{dn}{dx} \;=\; - n(x) \cdot 1/(kT) \cdot \frac{dE_C(x)}{dx}$$

- The slope of the conduction band comes directly from the spatially varying electric potential $V(x)$; to convert the electric potential to the absolute energy of a charge carrier, the elementary charge e is needed as an additional factor. From the p–n junction we know that the sign (i.e., the direction) of the electric field is identical to that of the slope of the conduction band. Thus, altogether we have

$$E(x) \;=\; - \frac{dV(x)}{dx} \;=\; 1/e \cdot \frac{dE_C(x)}{dx}$$

- Using this relation, the current balance from above becomes

$$e \cdot n(x) \cdot \mu \cdot E(x) \;=\; q \cdot D \cdot \frac{dn(x)}{dx}$$

$$e \cdot n(x) \cdot \mu \cdot E(x) \;=\; -q \cdot n(x) \cdot D \cdot e/(kT) \cdot E(x)$$

$$D \;=\; \mu kT/e$$

- In words: Equilibrium between diffusion currents and electrical currents for charged particles demands a simple, but far reaching relation between the diffusion constant D and the mobility μ.

Distinguishing again between electrons and holes gives as the final result the famous **Einstein–Smoluchowski relations**:

$$D_e \ = \ \frac{\mu_e \cdot kT}{e}$$

$$D_h \ = \ \frac{\mu_h \cdot kT}{e}$$

You may want to have a look at a different derivation in an advanced module.

Non-Equilibrium Currents

In the consideration above we postulated that there is *no net current flow*; in other words, we postulated *total equilibrium*. Now let's consider that there *is* some *net current flow* and see what we have to change to arrive at the relevant equations.

In order to be close to applications, we treat the *extrinsic* case and, since we do not assume equilibrium per se, we automatically do not assume that the carrier densities have their *equilibrium* values n_e(equ) and n_h(equ), but *arbitrary* values that we can express by some Delta to the equilibrium value. We thus start with

$$n_e \ = \ n_e(\text{equ}) + \Delta n_e$$

$$n_h \ = \ n_h(\text{equ}) + \Delta n_h$$

● Since carriers above the equilibrium density are often created *in pairs* we have for this *special, but rather common* case

$$\Delta n_e \ = \ \Delta n_h \qquad = \ \Delta n$$

$$\Delta n \ = \ n_e - n_e(\text{equ}) \ = \ n_h - n_h(\text{equ})$$

● *This is a crucial assumption!*

This allows us to concentrate on *one kind of carrier*, let's say we look at **n**-type **Si** with electrons as the majority carriers. We now *focus on holes* as the minority carriers since we always can compute the electron density n_e by

$$n_e \ = \ n_e(\text{equ}) + \Delta n_e \ = \ n_e(\text{equ}) + n_h - n_h(\text{equ})$$

● We now must consider **Fick's second law** or the **continuity equation** (it is the same thing for special cases, but the continuity equation is more general).

For the net (mobile) charge density ρ (which is the *difference* of the electron and hole density, $\rho = e \cdot (n_h - n_e)$, in *contrast to the total particle density*, which is the *sum*!) we have

$$\frac{\partial \rho}{\partial t} \ = \ - \text{div} \ (j_{\text{total}})$$

● With $j_{\text{total}} = j_e + j_h$ = sum of the electron and hole currents.

In the simplest form we have for the holes

$$\frac{\partial n}{\partial t} \ = \ - (1/e) \cdot \text{div} \ (j_h)$$

The factor **1/e** is needed to convert an electrical current j to a particle current j_{part} via $j = q \cdot j_{\text{part}}$, with $q = \pm e$. Here, as always, we have to pick the right sign for the elementary charge **e** (negative for electrons, positive for holes).

- This is simply the statement that the *charge is conserved*. It would be sufficient that no holes disappear or are created in any differential volume **dV** considered, i.e. **div j_h = 0**, to satisfy that condition.

But this is, of course, a condition that we know *not* to be true.

- In all semiconductors, we have constant generation and recombination of holes (and electrons) as discussed before. In *in equilibrium*, of course, the generation rate **G** and the recombination rate **R** are equal, so they cancel each other in a balance equation and need not be considered – since **div j_h = 0** is correct *on average*.
- We are, however, considering *non-equilibrium*, so we must primarily consider the recombination of the *surplus* minority carriers given by

$$\triangle n_h \; = \; n_h - n_h(\text{equ})$$

- Why? Because, as stated before, the generation essentially does *not* change, so it still balances against the recombination rate of the equilibrium density, and only the recombination rate of the surplus minorities, **$R^{\triangle}$ = [n_h − $n_h(\text{equ})$]/τ** needs to be considered (τ is the minority carrier life time).
- **$R^{\triangle}$ = [n_h − $n_h(\text{equ})$]/τ** is the rate with which carriers disappear by recombination, we thus must *subtract* it from the carrier balance as expressed in the continuity equation, and obtain

$$\frac{dn}{dt} = -\frac{n_h - n_h(\text{equ})}{\tau} - (1/e) \cdot \text{div}\,(j_h)$$

The current **j** can always be expressed as the sum of a *field* current and a *diffusion* current as we did above by

$$j_{h,\text{total}}(x) \; = \; e \cdot n(x) \cdot \mu \cdot E_x(x) - e \cdot D_h \cdot \frac{dn_h(x)}{dx}$$

Inserting this equation in our continuity equation yields

$$\frac{\partial n_h(x)}{\partial t} = -\frac{n_h(x) - n_h(\text{equ})}{\tau} - n_h(x) \cdot \mu \cdot \frac{\partial E(x)}{\partial x} - E(x) \cdot \mu \cdot \frac{\partial n_h(x)}{\partial x} + D \cdot \frac{\partial^2 n_h(x)}{\partial x^2}$$

- This is an *important*, if not so simple equation. It is not so simple, because the electrical field strength **$E(x)$** at **x** is a function of the carrier density **$n_h(x)$** at **x**, which is what we want to calculate! We have used the symbols for partial derivatives ("∂") to emphasize that it is in reality a three-dimensional equation.

We will now look at some applications of this equation.

Pure Diffusion Currents

Consider the minority carrier situation just outside of the space charge region of a biased **p–n junction**.

- If it is forwardly biased, a lot of majority carriers are flowing to the respective other side where they become minority carriers. They will eventually disappear by recombination, but the minority carrier density right at the edge of the space charge region will be larger than in equilibrium and will decrease as we go away from the junction.
- This is now shown in the illustration used before in the simple model of the **p–n** junction, but the *realistic* minority carrier situation is now included.
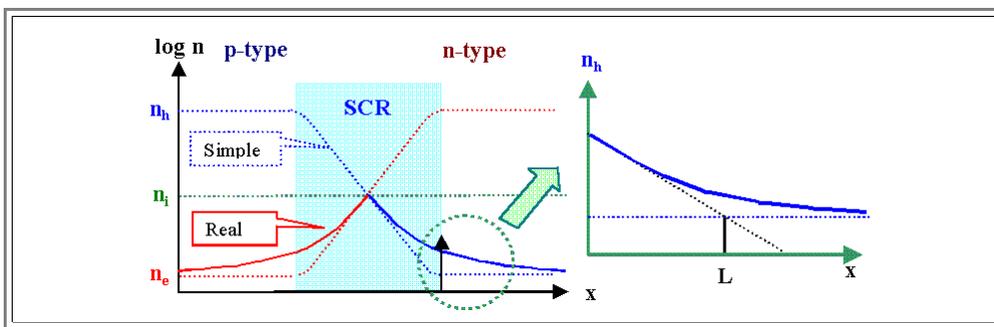
○ The region outside the space charge region, *while now showing a density gradient of the minority carrier density*, is essentially field free or at least has only a small electrical field strength.

▶ If we let $E_x = 0$ and consequently $\partial E_x(x)/\partial x = 0$, too, the current equation from above reduces to

$$\frac{\partial n_h}{\partial t} = -\frac{n_h - n_h(equ)}{\tau} + D \cdot \frac{\partial^2 n_h(x)}{\partial x^2}$$

○ Since $\partial n_h/\partial t = \partial[n_h(equ) + \Delta n_h]/\partial t = \partial \Delta n_h/\partial t$, and correspondingly $\partial^2 n_h(x)/\partial x^2 = \partial^2 \Delta n_h(x)/\partial x^2$, we have

$$\frac{\partial \Delta n_h}{\partial t} = -\frac{\Delta n_h}{\tau} + D \cdot \frac{\partial^2 \Delta n_h(x)}{\partial x^2}$$

▶ If we consider *steady state*, we have $\partial \Delta n_h/\partial t = 0$, and the solution of the differential equation is now mathematically easy.

▶ But how can steady state be achieved in practice? How can we provide for a *constant*, *non-changing* density of minority carriers *above* equilibrium?

○ For example by having a defined source of (surplus) holes at $x = 0$. In the illustration this is the (constant) hole current that makes it over the potential barrier of the **p–n** junction.

○ But we could equally well imagine holes generated by light a $x = 0$ at a constant rate. The surplus hole density then will assume some distribution in space which will be constant after a short initiation time - i.e. we have steady state and a simple differential equation:

$$D \cdot \frac{\partial^2[\Delta n_h(x)]}{\partial x^2} - \frac{\Delta n_h(x)}{\tau} = 0$$

▶ The solution (for a one-dimensional bar extending from $x = 0$ to $x = \infty$) is

$$\Delta n(x) = \Delta n_0 \cdot \exp - \frac{x}{L}$$

○ The length L is given by

$$L = \left(D_h \cdot \tau \right)^{1/2}$$

○ L is simply the diffusion length of the minority carriers (= holes in the example) as defined in the "simple" (but in this case accurate) introduction of life times and diffusion length.

○ This solution is already shown in the drawing above which also shows the direct geometrical interpretation of L.

▶ The important point to realize is that the *steady state* tied to this solution can only be maintained if the hole current at $x = 0$ has a constant, time independent value resulting from Fick's 1st law since we have no electrical fields that could drive a current.

○ This gives us

$$j_h(x = 0) = - e \cdot D \left. \frac{\partial \Delta n_h(x)}{\partial x} \right|_{x=0}$$

- By simple differentiation of our density equation from above we obtain

$$\left. \frac{\partial \Delta n_h(x)}{\partial x} \right|_{x=0} = - \frac{\Delta n_0}{L}$$

- Insertion into the current equation yields the final result

$$j_h(x = 0) = \frac{e \cdot D_h}{L_h} \cdot \Delta n_h(x = 0)$$

- The physical meaning is that the *hole part of the current* will decrease from this value as *x* increases, while *the total current* stays constant – the remainder is taken up by the electron current.

General Band-Bending and Debye Length

The *Debye length* and the *dielectric relaxation* time are important quantities for *majority* carriers (corresponding to the *diffusion length* and the *minority carrier life time* for *minority* carriers). Let's see why this is so in this paragraph.
- Both quantities are rather general and come up whenever density gradients cause currents that are counteracted by the developing electrical field.
- An alternative simple treatment of the Debye length can be found in a basic module.

Let us start with the Poisson equation for an arbitrary *one-dimensional* semiconductor with a varying electrostatic potential *V(x)* caused by charges with a density ρ(x) distributed somehow in the material. We then have

$$- \epsilon \cdot \epsilon_0 \cdot \frac{d^2 V(x)}{dx^2} = \epsilon \cdot \epsilon_0 \cdot \frac{dE(x)}{dx} = \rho(x)$$

- *E*(*x*) is the electrical field strength; always minus the derivative of the potential *V*.
- The charge ρ(*x*) at any one point can only result from our usual charged entities, which are electrons, holes, and ionized doping atoms. ρ(*x*) is always the *net* sum of this charges, i.e.

$$\rho(x) = e \cdot \left(n_h(x) + N_D^+(x) - [n_e(x) + N_A^-(x)] \right)$$

- The electrostatic potential *V* needed for the Poisson equation is now a function of *x* and shifts the conduction and valence band by the potential energy *qV* relative to some reference point for which one has *V = 0*. Since the band structure refers to the energy of electrons, we have that *q = –e* and thus may write

$$E_C(x) = E_C(V = 0) - e \cdot V(x)$$

$$E_V(x) = E_V(V = 0) - e \cdot V(x)$$

- Thereby, the Poisson equation becomes

$$- \epsilon \cdot \epsilon_0 \cdot \frac{d^2 V(x)}{dx^2} = \frac{\epsilon \cdot \epsilon_0}{e} \cdot \frac{d^2 E_C(x)}{dx^2} = e \left(n^h(x) + N_D^+(x) - [n^e(x) + N_A^-(x)] \right)$$

- If we now insert the proper equations for the four densities, we obtain a formidable differential equation that is of prime importance for semiconductor physics and devices, but not easy to solve.
- However, even if we could solve the differential equation (which we most certainly cannot), it would not be of much help, because we also a need a "gut feeling" of what is going on.

The best way to visualize the basic situation is to imagine a homogeneously doped semiconductor with a fixed charge density at its surface and no net currents (think of a *fictional insulating layer with infinitesimal thickness that contains some charge on its outer surface*).

- Carriers of the semiconducor thus can *not* neutralize the charge, and the surface charge will cause an electrical field which will penetrate into the semiconductor to a certain depth.
- This is the most general case for disturbing the carrier density in a surface-near region and thus to induce some *band-bending*.

There are *two* distinct major situations:

1. The surface charge has the *same polarity* as the majority carriers in the semiconductor, thus pushing them into the interior of the material.

- This exposes the ionized dopant atoms with opposite charge and a large **space charge layer** (**SCR**) will built up. This is also called the **depletion** case.
- The **SCR** is large because the dopant density is low and the dopant atoms *cannot move to the interface*. Many dopant atoms have to be "exposed" to be able to compensate the surface charge; the field can penetrate for a considerable distance.
- However: *In contrast* to what we learned about **SCRs** in **p–n** junctions, even for large fields (corresponding to large reverse voltages at a junction), the Fermi energy is E_F *still constant* (currents are not possible). The bands are still bent, however, this means that $E_C - E_F$ incrases in the direction toards the surface.
- If the majority carrier density then is becoming very small in surface-near regions (it scales with **exp [– ($E_C - E_F$)/ (kT)]** after all), the minority carrier density increases due to the mass action law until minority carriers become the majority – we have the case of **inversion**.

2. The surface charge has the *opposite polarity* as the majority carriers in the semiconductor, thus accumulating them at the surface-near region of the material.

- Then majority carriers can move to the surface near region and compensate the external charge. The field cannot penerrate deeply into the material.
- This case is called **accumulation**.

The situation is best visualized by simple band diagrams, we chose the case for **n**-type materials. The surface charge is symbolized by the green spheres or blue squares on the left.
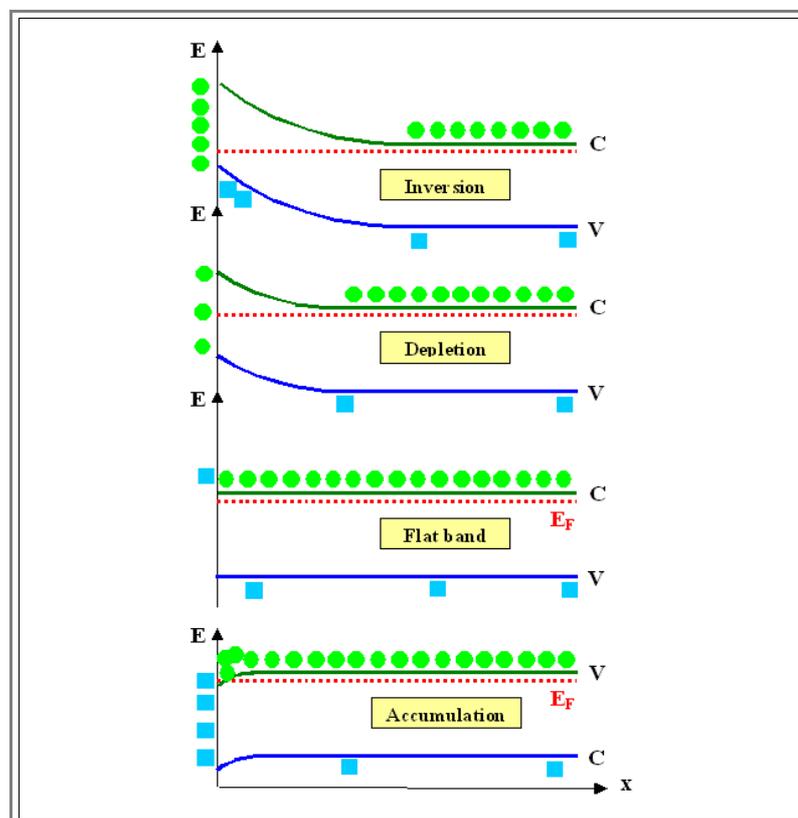
- Between depletion and accumulation must be the **flat-band** case as another prominent special case. This is not necessarily tied to a surface charge of zero (as shown in the drawing where a blue square symbolizes some positive surface charge), but for the external charge that compensates the charge due to intrinsic surface states.

We have [some idea](#) about the width of the space charge region that comes with the *depletion case*. But how wide is the region of appreciable band bending in the *case of accumulation*?

- Qualitatively, we know that it can be small - at least in comparison to a **SCR** - because the charges in the semiconductor compensating the surface charges are mobile and can, in principle, pile up at the interface

For the quantitative answer for all cases, we have to solve the [Poisson equation from above](#). However, because we cannot do that in full generality, we look at some special cases.

First we restrict ourselves to the usual case of *one* kind of doping – **n**-type for the following example – and temperatures where the donors are fully ionized, which means that the Fermi energy is well below the donor level or $E_D - E_F \gg kT$.

- We then have only *two* charged entities:

$$N_D{}^+ = N_D$$

$$n^e = N_{eff}{}^e \cdot \exp - \frac{E_C - E_F}{kT}$$

- This means in what follows we only consider the *majority carriers*.

The Poisson equation then reduces to

$$\frac{\epsilon \cdot \epsilon_0}{e} \cdot \frac{d^2 E_C(x)}{dx^2} = e \left(N_D - N_{eff}{}^e \cdot \exp - \frac{E_C(x) - E_F}{kT} \right)$$

- *And this, while special but still fairly general, is still not easy to solve.*

We will have to specialize even more. But before we do this, we will rewrite the equation somewhat.

- For what follows, it is convenient to express the band bending of the conduction band in terms of its deviation from the field-free situation, i.e. from $E_C{}^0 = E_C(x = \infty)$. We thus write

$$E_C(x) = E_C{}^0 + \Delta E_C(x)$$

- The exponential term of the Poisson equation can now be rewritten, we obtain

$$N_{eff}{}^e \cdot \exp - \frac{E_C(x) - E_F}{kT} = N_{eff}{}^e \cdot \exp - \frac{E_C{}^0 - E_F}{kT} \cdot \exp - \frac{\Delta E_C(x)}{kT}$$

- The first part of the right hand side gives just the electron density in a field-free part of the semiconductor, which – in our approximations – is identical to the density N_D of donor atoms. This leaves us with a usable form of the Poisson equation for the *case of accumulation*:

$$\frac{d^2 E_C}{dx^2} = \frac{d^2(\Delta E_C)}{dx^2} = \frac{e^2 \cdot N_D}{\epsilon \cdot \epsilon_0} \cdot \left(1 - \exp\left(-\frac{\Delta E_C}{kT} \right) \right)$$

ΔE_C characterizes the amount of band bending. We can now proceed to simplify and solve the differential equation by considering different cases for the sign and magnitude of ΔE_C.

- Unfortunately, this is one of the more tedious (and boring) exercises in fiddling around with the Poisson equation. The results, however, are of prime importance – they contain the very basics of all semiconductor devices.

We will do *one* approximative solution *here* for the most simple case of **quasi-neutrality** which will give us the all-important *Debye length*.

- The other cases can be found in advanced modules:

  - [Depletion](#)
  - [Inversion](#)
  - [Accumulation](#)
  - [Putting everything together](#)

Quasi-neutrality is the mathematically most simple case; it treats only *small deviations* from equilibrium and thus from charge neutrality.

- The condition for quasi-neutrality is simple: We assume $|\Delta E_C| \ll kT$.

- We then can approximate the exponential function by its *Taylor series* and stop after the second term. This yields

$$\frac{d^2(\Delta E_C)}{dx^2} = e^2 \cdot N_D \, \epsilon \cdot \epsilon_0 \cdot \frac{\Delta E_C}{kT}$$

- That is easy now, the solution is

$$\Delta E_C(x) = \Delta E_C(x=0) \cdot \exp - \frac{x}{L_{Db}}$$

- The solution defines L_{Db} = **Debye length** for *n-type semiconductors* = Debye length for electrons, we have

$$L_{Db} = \sqrt{\frac{\epsilon \cdot \epsilon_0 \cdot kT}{e^2 \cdot N_D}}$$

- Obviously the Debye length L_{Db} for *holes in **p**-type semiconductors* is given by

$$L_{Db} = \sqrt{\frac{\epsilon \cdot \epsilon_0 \cdot kT}{e^2 \cdot N_A}}$$

For added value, our solution also gives the field strength of the electrical field extending from the surface charges into the depth of the sample.

- Setting it to zero at the top of the valence band in the p-type material (as it is conventionally done), the electrostatic potential is related to the conduction band edge by $E_C(x) = E_g - e \cdot V(x)$. As discussed already above, the minus sign stems from the negative charge of an electron.

- Since the field strength $E(x)$ is minus the derivative of the electrostatic potential, we now have

$$E(x) = -\frac{dV(x)}{dx} = 1/e \cdot \frac{dE_C(x)}{dx} = -\frac{1}{e \cdot L_{Db}} \cdot \Delta E_C(x)$$

- Note that in the case of accumulation at the surface of an n-type semiconductor, $\Delta E_C(x)$ is negative, so the electric field comes out positive – in full agreement with the surface (at $x = 0$) being positively charged in this case.

The Debye length gives the typical length within which a *small* deviation from equilibrium in the *total charge density* – which for doped semiconductors is always dominated by the *majority carriers* – is relaxed or screened; in other words, it is no longer felt.

- L_{Db} is a *direct material parameter* – its definition contains nothing but prime material parameters (including the doping).

- For medium to high doping densities, it becomes rather small. The dependence of the Debye length on material parameters is shown in an illustration.

- The Debye length is also a prime material quantity in materials other than semiconductors - especially in ionic conductors and electrolytes (for which it was originally introduced). It also applies to metals, but there it is so small that it rarely matters.

The Debye length comes up in all kinds of equations. Some examples are given in the advanced modules dealing with the other cases of field-induced band bending

- *The Debye length is to <u>majority carriers</u> what the diffusion length is to minorities*. And just as the *diffusion length* is linked to the minority carrier lifetime τ, the *Debye length* correlates to a specific time, too, called the dielectric relaxation time τ_d.
- This will be the subject of the next paragraph.

Dielectric Relaxation Time

Let's start from the same situation that lead to the Debye length: A doped semiconductor, all dopants ionized, and some small disturbance in the charge equilibrium expressed as some small $\Delta\rho(x, t)$ somewhere, starting at some time t_0; i.e. we still assume <u>quasi-neutrality</u>.

- The Poisson equation now is extremely simple, we write it directly for the electrical field strength and have

$$\frac{d E(x, t)}{d x} = \Delta\rho(x, t) \, \epsilon \cdot \epsilon_0$$

We now want to find out about *how long it takes* to establish a steady state, so we need some expression for $d(\Delta\rho)/dt$. The Poisson equation won't help because it does not explicitly contain the time dependence.

- But simply using the <u>continuity equation</u> for the relevant charge density $\Delta\rho$ provides a $d(\Delta\rho)/dt$ term. Since we are treating quasi-neutrality, we neglect all terms with *gradients* in the carrier density (this will turn out to be fully justified).
- Since the only relevant current is the drift curent $j(x) = \sigma \, E(x) = \rho \cdot \mu \cdot E(x)$, this leaves us with the following *continuity equation*

$$\frac{\partial(\Delta\rho)}{\partial t} = -\rho \cdot \mu \cdot \frac{\partial E(x)}{\partial x}$$

Inserting dE/dx from the <u>Poisson equation</u> gives

$$\frac{\partial(\Delta\rho)}{\partial t} = -\frac{\rho \cdot \mu}{\epsilon \cdot \epsilon_0} \cdot \Delta\rho$$

- ρ is the total carrier density, we can write it as $\rho = \rho_0 + \Delta\rho \approx \rho_0$ since we have quasi neutrality; μ, as always, is the mobility of the carrier in question.

This is a differential equation for $\Delta\rho(x, t)$ with the simple solution

$$\Delta\rho(x, t) = \Delta\rho(x, 0) \cdot \exp{-\frac{t}{\tau_d}}$$

$$\tau_d = \frac{\epsilon \cdot \epsilon_0}{\mu \cdot \rho_0}$$

- With τ_d = **dielectric relaxation time** = another basic material constant for the <u>same reason</u> as the Debye length.
- The *dielectric relaxation time* tells us exactly what we wanted to know: How long does it take for the majority carriers to respond to a disturbance in the charge density.

While this definition of some special time is of some interest, but not overwhelmingly so, the situation gets more exciting when we consider relations between our basic material constants obtained so far:

- Since $\mu \cdot \rho = \sigma$, the <u>conductivity of the material</u> (for the carriers in question), we have the simple and fundamental relation

$$\tau_d = \frac{\epsilon \cdot \epsilon_0}{\sigma}$$

Now let's see if there is a correlation to the Debye length:

- We use the Einstein relation $D = \mu(kT/e)$, the Debye length definition ($L_{Db} = \{(\epsilon \cdot \epsilon_0 \cdot kT)/(e \cdot \rho)\}^{1/2}$, pluck it into the definition of the dielectric relaxation time (again replacing $e \cdot N_D$ by ρ) and obtain

$$\tau_d = \frac{L_{Db}^2}{D}$$

$$L_{Db} = \sqrt{D \cdot \tau_d}$$

- This is exactly the same relation for the majority carriers between a *characteristic time constant* and a *length* as in the case of the minority carriers where we had the minority lifetime τ and the correlated diffusion length L.
- The physical meaning is the same, too. In both cases the times and lengths give the numbers for how fast a deviation from the carrier equilibrium will be equalized and over which distances small deviations are felt.

This merits a few more thoughts.

- If the carrier density is high, τ_d is in the order of *picoseconds* and L_{Db} extends over *nanometers*. Any deviation from equilibrium is thus almost instantaneously wiped out, or, if that is not possible, contained within a very small scale.
- And this is the regular situation for *majority* carriers. The few minority carriers always present in the semiconductor, too, can be safely neglected.

For *minority* carriers, however, the situation is *entirely different*.

- Their density is very small; τ_d and L_D consequently are no longer small.
- Moreover, whatever disturbance occurs in the density of *minorities*, there are plenty of majorities that can react very quickly (with their τ_d) to the electrical field always tied to a $\Delta\rho_{min}$.

The majority carriers are always attracted to the minorities and thus will quickly surround any excess minority charge with a "cloud" of majority carriers (which is called *screening*), essentially compensating the electrical field of the excess minorities to zero.

- They will, of course, eventually remove the excess charge by recombination, but that takes *far longer* than the time needed to do the screening.
- Since the electrical field is now zero, the excess charge cannot disappear or spread out by field currents – only spreading by diffusion in the density gradient (which is automatically introduced, too) is possible.

But this is exactly the process that we have neglected in this discussion (we had all density gradients in the continuity equation set to zero!).

- *Dielectric relaxation* (i.e. the disappearance of charge surpluses driven by electrical fields) is thus not applicable to minority carriers. Charge equilibration there is driven by diffusion - which is a much slower process!
- This then justifies the simple approach we took before, where we only considered the diffusion of minorities and did not take into account the majority carriers.