Solution to Exercise 2.1-2

Derive numbers for v_0, v_D, τ, and *l*

First Task: Derive a number for $\mathbf{v_0}$ (at room temperature). We have:

$$v_0 = \left(\frac{3kT}{m}\right)^{1/2} = \left(\frac{3 \cdot 8.6 \cdot 10^{-5} \cdot 300}{9.1 \cdot 10^{-31}} \; \frac{\text{eV} \cdot \text{K}}{\text{K} \cdot \text{kg}}\right)^{1/2} = 2.92 \cdot 10^{14} \cdot \left(\frac{\text{eV}}{\text{kg}}\right)^{1/2}$$

- The dimension "square root of **eV/kg**" does not look so good - for a velocity we would like to have **m/s**. In looking at the energies we equated kinetic energy with the classical dimension $\mathbf{kg \cdot m^2/s^2 = J}$ with thermal energy $\mathbf{k}T$ expressed in **eV**. So let's convert **eV** to **J** (use the link) and see if that solves the problem. We have $\mathbf{1\ eV = 1.6 \cdot 10^{-19}\ J = 1.6 \cdot 10^{-19}\ kg \cdot m^2 \cdot s^{-2}}$, which gives us

$$v_0 = 2.92 \cdot 10^{14} \cdot \left(\frac{1.6 \cdot 10^{-19}\ \text{kg} \cdot \text{m}^2}{\text{kg} \cdot \text{s}^2}\right)^{1/2} = 1.17 \cdot 10^5\ \text{m/s} = 4.21 \cdot 10^5\ \text{km/h}$$

Possibly a bit surprising - those electrons are no sluggards but move around rather fast. Anyway, we have shown that a value of ≈ $\mathbf{10^4}$ **m/s** as postulated in the backbone is really OK.

- Of course, for $T \to 0$, we would have $v_0 \to 0$ – which should worry us a bit???? If instead of room temperature (**T = 300 K**) we would go to **1200 K**, let's say, we would just double the average speed of the electrons.

Second Task: Derive a number for τ. We have:

$$\tau = \frac{\sigma \cdot m}{n \cdot e^2}$$

First we need some number for the concentration of free electrons per $\mathbf{m^3}$. For that we complete the table given, noting that for the number of atoms per $\mathbf{m^3}$ (i.e, the atomic density) we have to divide the density by the atomic weight.

Atom	Density [$kg \cdot m^{-3}$]	Atomic weight [$1.66 \cdot 10^{-27}$ kg]	Conductivity σ [$10^7\ \Omega^{-1} \cdot m^{-1}$]	Atomic dens. [$10^{28}\ m^{-3}$]
Na	970	23	2.4	2.54
Cu	8,920	64	5.9	8.40
Au	19,300	197	4.5	5.90

- So let's take $\mathbf{5 \cdot 10^{28}\ m^{-3}}$ as a good order of magnitude guess for the number of atoms in a $\mathbf{m^3}$, and for a first estimate some average value $\sigma = \mathbf{5 \cdot 10^7\ \Omega^{-1}\ m^{-1}}$. We obtain

$$\tau = \frac{5 \cdot 10^7 \cdot 9.1 \cdot 10^{-31}}{5 \cdot 10^{28} \cdot (1.6 \cdot 10^{-19})^2} \; \frac{\text{kg} \cdot \text{m}^3}{\Omega \cdot \text{m} \cdot \text{A}^2 \cdot \text{s}^2} = 3.55 \cdot 10^{-14} \; \frac{\text{kg} \cdot \text{m}^2}{\text{V} \cdot \text{A} \cdot \text{s}^2}$$

Well, somehow the whole thing would look much better with the unit **s**. So let's see if we can remedy the situation.

- Easy: volt times ampere equals *watt*, which is power, i.e. energy per time, with the unit $\mathbf{J \cdot s^{-1} = kg \cdot m^2 \cdot s^{-3}}$. Insertion yields

$$\tau = 3.55 \cdot 10^{-14} \frac{\mathrm{kg \cdot m^2 \cdot s^3}}{\mathrm{kg \cdot m^2 \cdot s^2}} = 3.55 \cdot 10^{-14}\ \mathrm{s} = 36\ \mathrm{fs}$$

The backbone thus is right again. The scattering time is in the order of femtoseconds, which is a short time indeed. Since all variables enter the equation linearly, looking at somewhat other carrier densities (e.g. more than **1** electron per atom) or conductivities does not really change the general picture very much.

Third Task: Derive a number for $\mathbf{v_D}$. We have (for a field strength $\boldsymbol{E} = \mathbf{100\ V/m = 1\ V/cm}$):

$$|v_D| = \frac{E \cdot e \cdot \tau}{m} = \frac{100 \cdot 1.6 \cdot 10^{-19} \cdot 3.55 \cdot 10^{-14}}{9.1 \cdot 10^{-31}} \frac{\mathrm{V \cdot C \cdot s}}{\mathrm{m \cdot kg}} = 6.24 \cdot 10^{-1} \frac{\mathrm{V \cdot A \cdot s^2}}{\mathrm{m \cdot kg}}$$

$$= 6.24 \cdot 10^{-1} \frac{\mathrm{kg \cdot m^2 \cdot s^2}}{\mathrm{m \cdot kg \cdot s^3}} = 6.24 \cdot 10^{-1}\ \mathrm{m/s} = 624\ \mathrm{mm/s}$$

This is somewhat larger than the value given in the backbone text.

- However - a field strength of **1 V/cm** applied to a *metal* is huge! Think about the current density ***j*** you would get if you apply **1 V** to a piece of metal **1 cm** thick.
- It is actually $\boldsymbol{j} = \sigma \cdot \boldsymbol{E} = 5 \cdot 10^7\ \Omega^{-1}\ \mathrm{m}^{-1} \cdot 100\ \mathrm{V/m} = 5 \cdot 10^9\ \mathrm{A/m^2} = 5 \cdot 10^5\ \mathrm{A/cm^2}$!
- For a more "reasonable" current density of $\mathbf{10^3\ A/cm^2}$ we have to reduce ***E*** hundredfold and then end up with $|v_D| = \mathbf{6.24\ mm/s}$ – and that is slow indeed!

Fourth Task: Derive a number for ***l***. We have:

$$l = 2 \cdot v_0 \cdot \tau = 2 \cdot 1.17 \cdot 10^5 \cdot 3.55 \cdot 10^{-14}\ \mathrm{m} = 8.31 \cdot 10^{-9}\ \mathrm{m} = 8.31\ \mathrm{nm}$$

- Right again! If we add the comparatively miniscule $\mathbf{v_D}$, nothing would change.
- Note, however, that the last equation does NOT mean that decreasing the temperature would lower ***l*** to eventually zero! Why? Because τ isn't a constant but scales with the conductivity – look at the starting equation of the second task! And since the conductivity increases at lower temperatures, so does the mean free path.