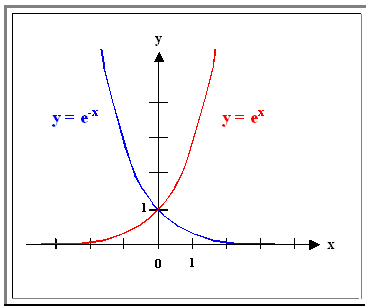


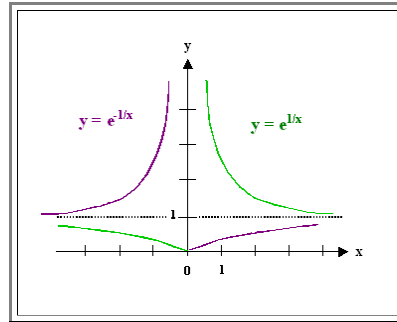
Working with Exponentials and Logarithms

Basics

First, the graphical representation of the most important exponential curves (see [math script](#) as well)



- The typical curves everybody should know. The blue curve in the first quadrant (positive x values) corresponds to the *energy dependence* of the ubiquitous **Boltzmann factor** $\exp - (E/kT)$



- Slightly more tricky. Note that the purple branch in the 1. quadrant corresponds to the *temperature dependence* of the ubiquitous Boltzmann factor $\exp - (E/kT)$

The inverted functions, e.g. $y = \ln x$ are easily pictured, too; below the $y = \ln x$ and the $y = \ln (1/x)$ functions are shown.



- The graphs, in case you forgot, also illustrate some basic algebraic relations, e.g.

- $e^{-x} = 1/e^x$
- $\ln x = -\ln (1/x)$

The essential identities are

$$e^x = \frac{1}{e^{-x}} \quad (e^x)^y = e^{x \cdot y}$$

$$e^x \cdot e^y = e^{x+y} \quad (e^x)^{1/y} = e^{x/y}$$

$$\frac{e^x}{e^y} = e^{x-y} \quad e^{\ln x} = x$$

$$\ln (x \cdot y) = \ln x + \ln y$$

$$\ln \frac{x}{y} = \ln x - \ln y$$

$$\ln x^y = y \cdot \ln x$$

Here some approximations.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\ln (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$$\ln (1-x) = - \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \right)$$

While many equations contain exponential terms of some variable which "disappear" if you substitute the **ln** of the variable (as in the **Arrhenius plot**), we mostly prefer the **lg** of some observable quantity to the **ln**. As an example, plotting the **vacancy concentration** c_v in an Arrhenius plot would be straight forward with the **ln**:

- On order to get a straight slope we have to switch to new variables according to

$$c_V = A \cdot \exp - \frac{H^F}{kT}$$

$$\ln c_V = \ln A - \frac{H^F}{kT} \cdot \frac{1}{T}$$

$$y = \ln A - \frac{1}{T} \cdot x$$

for

$$y = \ln c_V \quad x = \frac{1}{T}$$

● For y and x we get straight line with slope $-H^F/k$ and intercept = A

▮ What do we have to do if we want to plot $\lg(c_V)$ instead of $\ln(c_V)$?

● We have to multiply everything with $\lg e = 0,4342..$, because

$$\lg x = (\lg e) \cdot (\ln x) \quad 1)$$

● We obtain

$$\lg(c_V) = 0,434 \cdot \ln A - 0,434 \cdot \frac{H^F}{k} \cdot \frac{1}{T}$$

▮ 1) If this puzzles you, consider

● We postulate the equality

$$\lg x = M \cdot \ln x$$

● and want to find a value for M . Raising everything to the power of 10 gives

$$10^{\lg x} = x = 10^{M \cdot \ln x}$$

● This equation can only be fulfilled if

$$10^M = e$$

● because then we have $e^{\ln x} = x$ as required, giving $M = \lg e$