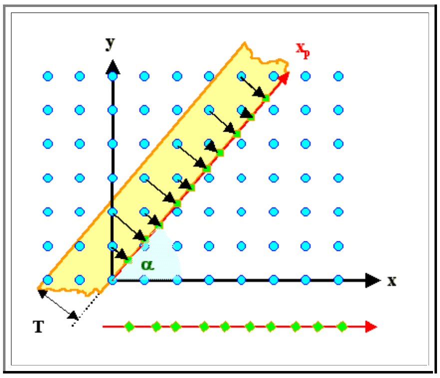
Quasikristalle II

This module will be in English because it is also used for other Hyperscripts.

Advanced

- One of the more exciting (or frightening, depending on one's perspective to science) developments in understanding quasicrystals was the insight that quasicrystals can be constructed by projecting a *perfect* lattice in *6-dimensional space* onto a properly chosen **3**-dimensional subspace (which is the kind of space you and I know).
  - The gist of this outrageous statement is actually far easier to understand than it appears at first. For that we look at a simple analogue: We project a *perfect **2**-dimensional lattice* on a ***1**-dimensional subspace*. How this is done is shown in the figure below.



  - Starting from a simple **2**-dimensional cubic lattice, we draw a straight line $\mathbf{x_p}$ at an angle α to the **x**-direction. The decisive point is that **tan** α must be an *irrational* number, e.g. $\mathbf{(5)^{1/2}}$ or **π/3** or whatever. This makes sure that $\mathbf{x_p}$ will *not* touch another lattice point *ever*.
  - We than define an area (yellow) by drawing a line parallel to $\mathbf{x_p}$ at some distance **T** , which can be an arbitrary number.
  - Some lattice points will now be found within the yellow area, we project their position onto $\mathbf{x_p}$.
- The sequence of points obtained that way (shown at the bottom as the sequence of green "diamonds") is - by definition if you think about it - aperiodic; it will *never repeat*.
  - But it is not completely random either. There are only two different distances between points, their sequence just does not follow a periodic pattern.
  - We have actually produced a *one-dimensional quasicrystal*.
- Now lets take a *six-dimensional space* and construct a cubic primitive lattice. No mathematician has the slightest problem doing that - you simply get a "hypercube" elementary cell with **64** corners and so on.
  - Now lets take a regular three-dimensional space. We make sure that the three-dimensional space is oriented relative to the six-dimensional space in such a way, that the six base vectors of the hypercube are projected onto the three-dimensional space with the fivefold symmetry of an *icosahedra*. (An icosahedra is one of those regular "eders" with triangular faces, where always five triangles sort of group around an axis with five-fold symmetry.
  - Interestingly, instead of some **tan** α the number $\mathbf{N = [1 + (5)^{1/2}]}$ appears - which is the "magic number of the **golden ratio**". This may or may not mean something special.
  - Now you define some neighborhood around your three-dimensional space and start projecting - you will get the exact arrangement of atoms in a real three-dimensional quasicrystal.
- Don't worry if you can not imagine all this - nobody can.
  - Only worry if you can't see that there is a sophisticated, but nevertheless rather clear-cut mathematical procedure of how to construct a three-dimensional point sequence by all this hypercube projection stuff.
  - If you did not worry so far, you should now, pondering the question: *What does it mean*?
- Again, who knows for sure? But we do know that there is some meaning. Lets just look at *one* example.
  - In three-dimensional quasicrystals we find some entities that look and behave exactly like dislocations (a one-dimensional defect in three-dimensional *lattices*). These whatever-they-are entities come into being after some deformation like *real* dislocations, they move through the quasicrystal like *real* dislocation, they look like *real* dislocations in the electron microscope - but they simply cannot be *real* dislocations because *real* dislocations can *only* exist in *periodic* lattices.
  - Turns out they are dislocations in the *six-dimensional periodic lattice* - no problem to define dislocations there. What we see is sort of what is left in the three-dimensional space where atoms live (not to mention you and me).
  - And while *real* dislocations are always characterized by their Burgers vector - just a regular vector with three components - the quasicrystal sort-of-dislocations need a vector with *six* components for their characterization.

- Mercifully enough, it turns out that the particulars of the projection scheme always allow to decompose the *one* six-dimensional Burgers vector into *two* three dimensional ones. These two regular vectors have a precise meaning as to what this dislocation-liek thing does to the quasicrystal when it moves through it.

All in all - quasicrystals, quite unexpectedly, link materials science with rather involved and rather unexpected math. It will be exciting to witness what will come from all this during the next **10** or **20** years.