The cgs System of Units

General Remarks

Basics

- Physicists love (one of) the **cgs** systems and stubbornly keep using it - even so it is "forbidden". Take the Barrett for example, a rather recent book - you will find *cgs units*. Here we will see *why* the **cgs** system still holds a lot of attraction, and how to convert **cgs** units to **SI** units.
- As long as you just use the basic units *length*, *mass* and *time*, it really does not matter much if you work with the **m, kg, s**, i.e. in **SI** units, or with **cm, g and s** - ***cgs** units*.
  - Engineers at your (future) level of sophistication simply can do the conversions without having to be taught *and* without making mistakes.
- A (big) problem, however, emerges as soon as you add some basic unit for *electricity*.
  - The **SI** system chose the electrical current with the unit Ampere (**A**) - and that is all there is. Like with the meter, you now need some arbitrary, but generally accepted *reference* that defines **1 A** (and at the same time gives a recipe how it will be measured).
  - For the meter, we originally picked **1/10 000 000** part of the circumference of the earth and deposited that as a **Pt-Ir** piece in Paris. Later it was replaced by something better, but the general idea is the same.
  - Likewise for the kilogram and the second - but what do we take for the Ampere?
- Well, something not all that smart (from the viewpoint of physicists and practicing engineers):
  - **1 Ampere** is the magnitude of a constant electrical current, which, if running through two infinitely long parallel wires with negligeable circular cross sections kept at a distance of **1 m** in vacuum, produces a force of exactly **2 · 10^{-7} N** per meter of wire.
  - This definition also defines **charge** in Coulomb, by simply equating
Charge **[C]** = *(Ampere · Time)* **[As]**, i.e.
1 C = 1 As.
  - Who needs forces between wires? We rarely do. What we *do* need a lot, however, are forces between *charges*. Lets see what we get for this.
- We start from the universal relation between an electrical field ***E*** and the force ***F*** that a charge ***Q*** experiences in said field (with the very important corollary that the field produced by ***Q*** is *not* added to the field already there! Asking "why" leads into really deep water, cf. chapter 28 (Vol. II) in the Feynman lectures (perfectly understandable to undergraduates!))
  - We have $F = E \cdot Q$, and since ***F*** and ***Q*** are already defined, this equation *defines* ***E***. Enter a number for ***F*** and ***Q*** and you get a number for ***E***. You akso get a unit:
[E] = N/As
  - So far we have no problem. But now we look at the force on a (point charge) ***q*** that results from another point charge ***q'*** by *first* computing the field of one of the charges and than applying the formula from above.
- Lets take a "point charge" ***q*** and the simple statement (from Maxwell) that the *electrical flux density* Π through a closed surface around a charge is proportional to the charge inside the surface. We take it as *proportional*, because the numerical value of the proportionality constant will depend on the choice of units, which we try to unravel.
  - Take a sphere with radius ***r*** around ***q*** and you have

$$\Pi \;=\; \oint E\,df \;=\; 4\pi r^2 \cdot |E| \;=\; \gamma \cdot q$$

  - With γ = proportionality constant and ***df*** = incremental area element. This gives us for the numerical value of the electrical field strength at a distance ***r*** from a point charge ***q***

$$E \;=\; \frac{\gamma \cdot q}{4\pi \cdot r^2}$$

  - Now we know the field at some distance ***r*** from the charge, and therefore the force ***F*** on a charge ***q'***; we have

$$F = q \cdot q' \cdot \frac{\gamma}{4\pi \cdot r^2}$$

- This is completely general, as long as we make no assumptions for γ.

OK, *now* lets discuss the possibilities for the value of γ.

- If we use **SI** units, we have *no choice*: We already have definitions for *force*, *charge* and the *electrical field* ***E*** in **SI** units the numericla value of γ is determined. We simply have (writing γ as $\gamma = 1/\epsilon_0$

$$F = q \cdot q' \cdot \frac{1}{4\pi \cdot \epsilon_0 \cdot r^2}$$

- With $\epsilon_0 = 8{,}859 \cdot 10^{-12}$ **As/Vm =** proportionality constant between the charge q inside some body and the total flux Π through the surface of that body, or

$$\Pi = \oint Edf = \frac{q}{\epsilon_0}$$

- Forgetting for a moment that we *must* use **SI** units, we could make life a lot easier by simply defining $\gamma := 4\pi$, and presto, we have a Coulomb law as you find it quite often in text books, that can be written most simply as

$$F = \frac{q \cdot q'}{r^2}$$

- It's easy to see why people like that - you simply save a lot of boring writing.

However, since you already have defined lengths and forces somehow (in the **cgs system** they were given in "**dyn**" (**1 dyn = 1 gm/s^2 = 10^{-5} N**)), *you are now making a statement as to how you measure charge*. The easiest thing to do is to define charge in such a way that we get a unit of force (**= 1 dyn**) if two units of charge (**= ???**) are one unit of distance (**= 1 cm**) apart.

- This means we take the numerical value of q and q' to be $q = q' = 1$ **[?]**, and the distance r to be **1 cm**.
- Your force must come out to be *one dyn*; we have
  F = 1dyn = 1 gcm/s^2 = 1 [?]·[?]/cm^2
  which gives our unit of charge to be $[q]^2 = g \cdot cm^3/s^2$, or

$$[q] = g^{1/2} \cdot cm^{3/2} \cdot s^{-1}$$

A bit strange, but who cares. You simply call it an "*electrostatic unit* (***esu***)" (**ESL** in German; for "elektrostatische Einheit").

- Essentially, instead of defining the unit of current (**= 1 A**) by some force law, you define a *charge* by some other (but more frequently used) force law. Add it to your **cgs** basic units as the essential required input for electricity, and you have the "*electrostatic*" **cgs** system, sometimes called **CGSF** system (the "**F**" stands for "*Franklin*, **[Fr]**", which was the name given to the unit of charge); i.e. $1\ Fr = 1\ g^{1/2} \cdot cm^{3/2} \cdot s^{-1}$.
- However, nobody uses the name "*Franklin*" anymore, you just call it electrostatic unit charge, or **esu**, or whatever.
- Look at this copy form the Feynman lectures, to see how pissed people become with the **SI** convention!

There is a *physical* reason for being able to write the dipole potential in the form of Eq. (6.16). Suppose we have a point charge q at the origin. The potential at the point P at (x, y, z) is

$$\phi_0 = \frac{q}{r}.$$

(Let's leave off the $1/4\pi\epsilon_0$ while we make these arguments; we can stick it in at the end.) Now if we move the charge $+q$ up a distance Δz, the potential at P will change a little, by, say, $\Delta\phi_+$. How much is $\Delta\phi_+$? Well, it is just the amount that the potential *would* change if we were to *leave* the charge at the origin and move P *downward* by the same distance Δz (Fig. 6–5). That is,

$$\Delta\phi_+ = -\frac{\partial\phi_0}{\partial z}\Delta z,$$

where by Δz we mean the same as $d/2$. So, using $\phi = q/r$, we have that the potential from the positive charge is

$$\phi_+ = \frac{q}{r} - \frac{\partial}{\partial z}\left(\frac{q}{r}\right)\frac{d}{2}. \qquad (6.17)$$

Applying the same reasoning for the potential from the negative charge, we can write

- Now, and this brings in a lot of confusion, instead of a *special measure for charge* neede to make the **cgs** system "electric", you also could add something else "electrical" - and out come many *different* kinds of **cgs** unit systems.
    - But that is only for freaks (if you run across it and cannot avoid it - look it up in a really good handbook).
- Here we will only give some "translation" table, converting quantities from one system to the other. It is more tricky than it looks like!
- *Conversion of charge*.
    - We must ask ourselves: How many *esu per s* do we have to run through our wires from above to produce a force of **2 · 10 $^{-7}$ N** per meter of wire? This then must be **1 C**.
- Problem: What is the force between two wires running some current?
    - You see again, why most of us prefer the **cgs** system: We all know the Coulomb law by heart, but the force between current-carrying conductors???
    - OK, here is how you start: The **Lorentz law** tells us that the force $\underline{F}$ on some charge q moving with the speed $\underline{v}$ in a magnetic field $\underline{B}$ is

$$\underline{F} = q \cdot \underline{v} \times \underline{B}$$

- Next, the magnetic field around a wire with a current I running through it is

$$B = ½\pi\epsilon_0 c^2 \cdot \frac{I}{r}$$

- With c = (vacuum) speed of light.
- But what is the resulting formula for the two-wire-arrangement needed for defining I ?

- Interestingly enough, several standard text books on electrodynamics do *not* give you the formula directly - of course, it is no big deal to derive it yourself.
    - Still, it shows that the magnetic force formula is far less important than the Coulomb law. Without going into the details, lets just say that the factor for converting the charge from **cgs** to **SI** and back, is **c/10** or **10/c**, respectively.
    - Here are a few conversions:

Quantity	**cgs**	**Unit**	**=**	**SI**	**Unit**	**Conversion**
Charge	1	$cm^{3/2} \cdot g^{1/2} \cdot s^{-1}$ (esu)	=	$3{,}3356 \cdot 10^{-10}$	A·s = C	multiply by **|c|/10 = $3{,}3356 \cdot 10^{-10}$** **|c|** = number for vacuum speed of light in cm/s
	$2{,}998 \cdot 10^{9}$ esu	esu	=	1		
Elementary charge e	$4{,}8033 \cdot 10^{-10}$	esu	=	$1{,}602 \cdot 10^{-19}$	A·s = C	
Current I	1	$cm^{3/2} \cdot g^{1/2} \cdot s^{-2}$	=	$3{,}3356 \cdot 10^{-10}$	A	multiply with c/10

Voltage U	1	$cm^{1/2} \cdot g^{1/2} \cdot s^{-1}$	=	$2{,}9979 \cdot 10^2$	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$ = V	multiply with $10^{-8} \cdot |c|$

- And so on. For about **15 - 20** more conversions, consult some handbook.
  - The real problem, however, is not conversion. The real problem is: *The formulas are different!*
  - But first let's just look at our Coulomb attraction between point charges again, and see if the formulas really work

cgs	**SI**
$F = q \cdot q'/ r^2$	$F = q \cdot q' / 4\pi\epsilon_0 \cdot r^2$
Use **[esu]** for ***q***, **[cm]** for ***r*** ***F*** will be in **[dyn]**.	Use **[C]** for ***q***, **[m]** for ***r*** ***F*** will be in **[N]**.
Check: Calculate the force for ***r* = 0,1 nm = 10^{-8} cm = 10^{-10} m**, and ***q*, *q'*** = elementary charge	
$F = \dfrac{(4{,}803 \cdot 10^{-10})^2}{1^2}\ [esu^2 \cdot 10^{16} \cdot cm^{-2}]$ $F = 2{,}307 \cdot 10^{-3}\ [cm \cdot g^1 \cdot s^{-2}]$ $= 2{,}275 \cdot 10^{-3}$ **dyn** $= 2{,}275 \cdot 10^{-8}$ **N** $F = \ = 2{,}275 \cdot 10^{-3}$ **dyn** $= 2{,}275 \cdot 10^{-8}$ **N**	$F = \dfrac{(1{,}602 \cdot 10^{-19})^2}{4 \cdot 3{,}14 \cdot 8{,}854 \cdot 10^{-12} \cdot 1 \cdot 10^{-20}} \left[\dfrac{C^2}{A \cdot s \cdot V^{-1} \cdot m^{-1} \cdot m^2}\right]$ $F = \dfrac{2{,}566 \cdot 10^{-38}}{111.206 \cdot 10^{-32}} \dfrac{A^2 \cdot s^2}{A \cdot s \cdot V^{-1} \cdot m^1}$ Now we need **$1\ V = m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$** and get $F = \quad 2{,}307 \cdot 10^{-8} \quad m \cdot kg \cdot s^{-2} \cdot$ or $F = \ 2{,}307 \cdot 10^{-8}$ **N**

- Ok - it works well enough; just with some more numerics for the **SI** system.
  - So now you know: Whenever you see some expression relating to electrical forces, energies, or the like, *without an ϵ_0 or μ_0* in it, you are dealing with *cgs* units. And now you can convert to **SI** units with ease, if they appear in some book, *right?*
  - *Wrong*. Take the expression for the equilibrium of forces in Bohr's model from page **19** of the "Barrett", for example. It says in equations (2-4)
    $Ze^2/r = mv^2/ r$.
    So it must be given in **cgs** units (no particular statement is made).
  - So there should be a conversion or unit table somewhere, usually at the end of the books. And there it is, on p. 543, saying
    "Electronic charge, **e = $1{,}60 \cdot 10^{-20}$** ***emu*** **= $1{,}60 \cdot 10^{19}$ C**
- What the .... are ***emu's***? In cross word puzzles, emus appear as relatives of ostriches, but here it must be something else.
  - Yes - an "*emu*" is the electro*magnetic* charge unit; we have the *"magnetic"* ***cgs*** *system* here, it seems.
  - Now you look up your trusty handbook, (e.g. the "Physikalisches Taschenbuch" if you are a German) and find that the number given for charge, as measured in ***cgs*** *magnetic units*, is to be multiplied by **10** to get the **SI** coulombs, indeed, and that an "**emu**" is **1 $cm^{1/2} \cdot g^{1/2}$**.
- Great - but now pluck it into the formula, and things will be very wrong - by a factor **c^2**. The formula given requires **esu**'s, for **emu**'s, there must be a **c^2** somewhere. The "Barrett" simply got it wrong! Changing electric units changes the formulas!
- In other words, just switching from *esu* or *emu* to *C* will *not* do the trick and switch the *resulting force* from *dyn* to *N* in the electric world!

- While this is quite trivial on the one hand (we could have introduced conversions for the force, too), it simply means that if you want to keep some quantities (like the force) expressed in conventional units - length, mass and time - you must change your formulas, and complete them with the required **4π**, ϵ_0 (or alternatively $\mu_0 = 1/\epsilon_0 c^2$, and possibly other adornments as well.
- And while this conversion is always possible and not all that difficult to figure out, there is *plenty of room* for confusion and mistakes - consider the "Barrett example".

Here is a conversion table for the formulas. Whenever you encounter one of the quantities in the middle column in some **cgs** system formula, you replace it with the expression in the right hand column to obtain the formula in the **SI** system. But *remember*: You must also use the proper units!

Quantity	**cgs** (Gauss or electrostatic)	**SI**
Speed of light in vacuum	**c**	$(\mu_0\epsilon_0)^{1/2}$
Electrical field or potential, voltage	**E** **{Φ, V}**	$[(4\pi\epsilon_0)^{1/2}]\cdot E$ $\{\cdot\Phi, \cdot V\}$
Electrical flux density	**D**	$[(4\pi/\epsilon_0)^{1/2}]\cdot D$
Charge or current, current density, polarization	**q** **{I, j, P}**	$[1/(4\pi\epsilon_0)^{1/2}]\cdot q$ $\{\cdot I, \cdot j, \cdot P\}$
Magnetic induction (= magnetic flux density)	**B**	$[(4\pi/\mu_0)^{1/2}]\cdot B$
Magnetic field	**H**	$[(4\pi\mu_0)^{1/2}]\cdot E$
Magnetisation	**M**	$[(\mu_0/4\pi)^{1/2}]\cdot M$
Conductivity	σ	$[1/(4\pi\epsilon_0)]\cdot\sigma$
Dielectric constant	ε	ϵ/ϵ_0
Magnetic permeability	μ	μ/μ_0
Resistance or Impedance	**R** {Z}	$(4\pi\epsilon_0)\cdot R$ $\cdot\{Z\}$
Inductivity	**L**	$(4\pi\epsilon_0)\cdot L$
Capacity	**C**	$[1/(4\pi\epsilon_0)]\cdot C$

So here is what *you* do:

- *Use SI units* - even if it gives you an ulcer and you grind your teeth a lot. It will avoid many ulcers in the future.
- If you run across equations, numbers, relations, anything where you are not sure what kind of unit system is used - *be very careful*! Often it is best to pluck in some numbers and see if what you get makes any sense. Quite often, the result is so far off anything sensible (many orders of magnitude, like a factor c^2) that it just is clear that there is a confusion of units.
- But, on occasion, it is only a factor **4π** - be careful!