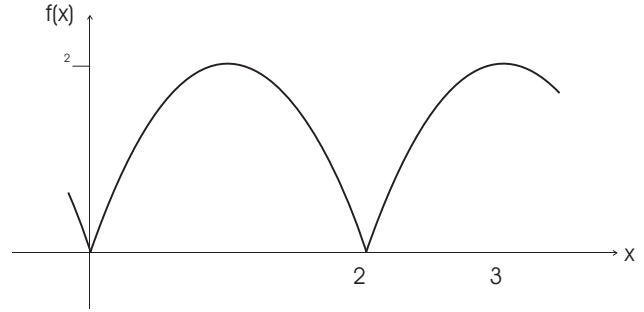


3.11.4 Example: Periodic parabolic function

$f(x) = x(2\pi - x)$, $0 \leq x \leq 2\pi$ periodic continuation



$$\frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} (2\pi x - x^2) dx = \frac{1}{2\pi} \left[\pi x^2 - \frac{1}{3} x^3 \right]_0^{2\pi} = \frac{2}{3} \pi^2$$

$$a_k = \frac{1}{\pi} \int_0^{2\pi} 2\pi x \cos kx dx - \frac{1}{\pi} \int_0^{2\pi} x^2 \cos kx dx$$

$$= \frac{1}{\pi} \left[2\pi \left(\frac{\cos kx}{k^2} + \frac{x \sin kx}{k} \right) - \left(\frac{2x}{k^2} \cos kx + \left(\frac{x^2}{k} - \frac{2}{k^3} \right) \sin kx \right) \right]_0^{2\pi}$$

$$\frac{1}{\pi} \left(-\frac{4\pi}{k^2} \right) = -\frac{4}{k^2}$$

$$\text{Thus: } f(x) = \frac{2}{3} \pi^2 - 4 \sum_{k=1}^{\infty} \frac{\cos kx}{k^2} \Rightarrow \text{Due to } 1/k^2 \text{ fast converging series}$$