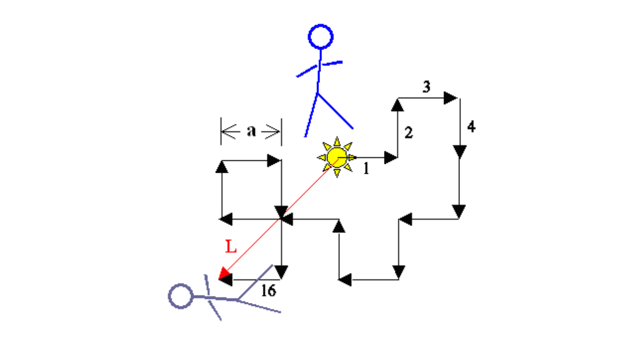
Diffusion

Science

2. Random Walk

Basics

- In the main text I made a lot of noise about "random walk". The text book paradigm for this always was the "**drunken sailor**", running around at random in two dimensions. In our enlightened times we must augment this by drunken cheerleaders to mainstream the genders in a politically correct way.
  - So imagine the drunken person of your choice to come out of the bar and to proceed in a haphazard way. To make it easier, we imagine that the drunken person makes steps of always the same length ***a***, and moves with equal likelihood forward, backward, left, or right. Maybe there is a chess board pattern on the plaza, and the drunken person tries to stay on the squares; whatever. The level of intoxination is such that the drunken person makes a total of ***N*** steps and then lies down and goes to sleep. The figure below illustrates this.



Slightly idealized walk of drunken person.

  - You follow a first individual and record the distance L_1 that your first specimen covered; the distance between the *start* position (exit of bar) and stop position (where the cheerleader beds down to sleep).
    Now you watch the next guy coming out of the bar; your specimen No. 2. You probably will find this guy in a competely different distance ***L*** - closer to the bar or further away - in comparison to the first one. You will never find any of them at a distance larger than $N \cdot a$ for obvious reasons, though.
    Now you do the same thing for No. 3, No 4, and a lot more. Then you determine the *average* distance covered by all those helpless people. This *average* distance we call the **diffusion length *L*(*N*, *a*)** of the random walk with step lenght ***a*** and number of steps ***N*** .
  - In contrast to an individual distance L_i, the average ***L*** is a well-defined quantity, just like the average income or the average distance people drive in a year. It is just very time consuming to *measure* it as described above. A good theory would be rather helpful.
- You will be delighted to hear that a good theory does exist. While it is not all that easy to derive the relevant equations, the final esult is extremely simple. Here it is:

$$L^2(\text{2-dim}) = 2 \cdot a^2 \cdot N$$

$$L^2(\text{3-dim}) = 3 \cdot a^2 \cdot N$$

$$L \approx a \cdot N^{1/2}$$

- If we forget about the square root of **1**, **2** or **3** for 1-dim., 2-dim. or 3-diminsional ramdom walks (the latter done by drunken sea gulls or sharks), the theory says unambigously that the diffusion length, the average distance between start and stop, is
  - proportional to the *distance* ***a*** covered in one step, and
  - proportional to the *square root* of the *number* of steps made.
- Great - but how about if the step length of the drunken animal is not the same at every step but varies? Then take ***a*** to be the *average* step length.
  You might come up with more questions like this but rest assured that the simple equation above covers every conceivable situation; we do not need to worry about details.

There is an immediate consequence of interest to sword smiths. The *total* distance or *path length* ***P*** covered by a random walker or flier is, of course, $P = N \cdot a$.

- This means that if you want to double the diffusion length, you have to walk a fourfold path length; a tenfold diffusion length increase involves a hundred fold distance, and so on. That means that if you want to diffuse some carbon into the outer layer of your steel for some case hardening, that you need to quadruple the time you keep it a high tempertures for doubling the thickness of the hardened layer.

Now let's do another simple experiment. You don't have to get up and get out, watching drunken sailors and cheer leaders stumbling around outside bars, you can do it right at home. All you need is a ruler and two coins.

- Put one coin in the middle of the ruler, e.g on the 8'' mark for a 16'' ruler. Flip the other coin, and move the first coin 1'' to the right for head, and 1'' to the left for tail.
  Write down the distance of the first coin from the starting position after you did 100 tosses and thus hundred left or right movements.
  Repeat this a hundred times or so, then calculate the average distance form the starting position for movements to *one* side - left or right.
  This is the diffusion length in a 1-dim. random walk. It should come out as predicted by the equation above.
  You can do even more. Produce a graph that shows how many coins ended up 1'' to one side, 2'' to one side and so on. That would be the *distribution function* of the distances covered.
- You must admit that this is far easier to do than following drunken people. Maybe a bit boring, but easy.
  Of course, if you know a bit about computer programming you can force your PC or notebook to do the work for you. Let the comoputer flip a digital coin and move a point on the screen one step to the left or right, accordingly. Actually, the computer won't mind to do that for a lot of points simultaneously and follow every one of them with total precision so you can get good statistics rather quickly
- If you are not so sure about writing computer programs you can take the easy way out and use the program two of my students (Niclas Köser und Sören Witt) have written for you. Here it is:

Random Walk Simulator

The program starts with a bunch of red points, all of which sit at the zero point of the "ruler"

- Pressing the "Run" bottom indiced the computer to flip a digital coin for every point and move it left or right. You can see that nicely by looking at the front runners.
- On top of the cloud of moving dots the graph mentioned above is constructed. Obviously, a Gaussian Bell curve develops with time - what else could it be?
- The program can also be used to simulate the (1-dim.) movement of real atoms that jump over some emegry barrier.

Back to

- [Diffusion]
- [1. Atomic Mechanisms of Diffusion]

On to

- [3. Phenomenological Modelling of Diffusion]
- [4. Experimental Techniques for Measuring Diffusion Parameters].
- [5. Diffusion in Iron]