## Units of Length, Area, and Volume

## Units General

To make life easier for everybody, the choice of units was taken away from you and me, not to mention your local Lords, Archdukes or Kings, more than 100 years ago, and given to scientists. As a result, everybody is now required to adhere strictly to the international standard system or SI units. Of course, the SI units are based on the metric system.
The SI system basically knows only seven basic units. Here they are:

| Quantity | Name | Unit |
| :--- | :--- | :--- |
| Length | Meter | m |
| Mass | Kilogram | kg |
| Time | Second | s |
| Electrical current | Ampère | A |
| Thermodynamic temperature | Kelvin | K |
| Amount of substance | Mol | mol |
| Luminous intensity | Candela | cd |

Let's forget about the "Luminous intensity" right away, and refer "Amount of substance" and "Thermodynamic temperature" to their links.
Time is not a problem because seconds, minutes, hours etc. are rather well known to all of us (and still nonmetric!).
Those of us who know what an electrical current is, know about Ampere (A) and I need not explain. If you personally are not always 100 \% sure if it should be Ampere (A) or Volt (V), you don't know a thing about electricity and I needn't explain either.
Mass is not too difficult either. In case of doubt you can go far with remembering that 1 kg are roughly 2 pounds. That leaves us with length and the general issue of how to deal with very small or very large numbers.
In this Hyperscript we need to deal with very small lengths. But for good measure, I also throw in very large lengths.
We deal with very small or very large numbers in two way. First and best by using "exponentials", i.e. by giving the potency of 10 . The potency is the exponent or the small raised number to the right of the " 10 ". It simply gives the number of zeros that you would have to write in the good, old-fashioned way

- $10^{6} \mathrm{~m}=1.000 .000 \mathrm{~m}=1$ million m
- $4,36 \cdot 10^{2} \mathrm{~m}=4,36 \cdot 100 \mathrm{~m}=436 \mathrm{~m}$.
- $9,460,730,472,580.8 \mathrm{~m}=9.460 \ldots \cdot 10^{15} \mathrm{~m}=9.46 . . \mathrm{Pm}=1$ light year

If the exponent is negative, it gives the number of zeros for the "one over the number":

- $10^{-6} \mathrm{~m}=(1 / 1,000,000) \mathrm{m}=1$ millionth $\mathrm{m}=1 \mu \mathrm{~m}=0,000001 \mathrm{~m}$.
- $4,36 \cdot 10^{-2} \mathrm{~m}=4,36 \cdot 1 / 100 \mathrm{~m}=4,36 / 100 \mathrm{~m}=0,0436 \mathrm{~m}$Second, we assign special names and abbreviations of those names to (mostly) orders of thousands. For example: "kilo" = thousand, "mega" = Million. For small numbers it's for example "micro" = millionth, or "nano" = billionth (american kind).
Heres is the list. Besides numbers some (small) relations are also given. The most common one is the percent (\%), but there are more, always called "parts per...".

| Large numbers |  |  | Small numbers |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Potency | Name | Symbol | Potency | Name | Symbol | As "parts per .." |
| $10^{24}$ | Yotta | Y | $10^{-1}$ | Dezi | d |  |
| $10^{21}$ | Zetta | Z | $10^{-2}$ | Zenti | c | \% <br> Percent |
| $10^{18}$ | Exa | E | $10^{-3}$ | Milli | m |  |
| $10^{15}$ | Peta | P | $10^{-6}$ | Micro | $\mu$ | ppm <br> million |
| $10^{12}$ | Tera | T | $10^{-9}$ | Nano | n | ppb <br> billion |
| $10^{9}$ | Giga | G | $10^{-12}$ | Pico | p | ppt <br> trillion |
| $10^{6}$ | Mega | M | $10^{-15}$ | Femto | $f$ | ppqt <br> quadrillion |
| $10^{3}$ | Kilo | k | $10^{-18}$ | Atto | a |  |
| $10^{2}$ | Hecto | h | $10^{-21}$ | Zepto | z |  |
| $10^{1}$ | Deca | da | $10^{-24}$ | Yocto | y |  |

## Nanometers

We shall be quite interested in the nanometer ( nm ), the billionth part of a meter, or $10^{-9} \mathrm{~m}$. You Americans may think of yards instead of meters here. 1 yard is roughly 1 m .

A nanometer is not a lot. Let's get a feeling for what it implies by looking at some small things you might relate to:

- The thickness of your hair is about 30 millionth of a meter $=30$ micrometers $=30 \mu \mathrm{~m}$. In nanometers that would be 30.000 nm .
- Small bacteria or cells of your tissue might be 1 millionth of a meter $=1 \mu \mathrm{~m}=1,000 \mathrm{~nm}$ in size. They are about the smallest things barely visible in a good light optical microscope at the highest magnification level.
- The transistors on a modern computer chip are about $0.1 \mu \mathrm{~m}$ or 100 nm in (lateral) size. That's about half as large as the wavelength of visible light. That's why you get all these colorful rainbow effects when you shine light on chips or just DVDs.
- The DNA molecule in your cell nuclei has a diameter of about 2.5 nm .
- An atom of whatever element you are considering is roughly about $0.2 \mathrm{~nm}-0.3 \mathrm{~nm}$ in size.
- The nucleus of the atoms you are considering is about one millionth of a nm or $10^{-6} \mathrm{~nm}$. It contains practically the complete mass of the atom. So an atom is essentially empty space on a small scale, just like the solar system on a large scale.


## Counting

No let's see how we count if we don't use the scientific abbreviations like mega = million
For some odd reason, different cultures count differently whenever they get to large numbers:

| Potency | Scientific Name | German <br> Name | American <br> Name |
| :--- | :--- | :--- | :--- |
| $10^{3}$ | Kilo | Tausend | Thousand |
| $10^{6}$ | Mega | Million | Million |
| $10^{9}$ | Giga | Milliarde | Billion |
| $10^{12}$ | Tera | Billion | Trillion |
| $10^{15}$ | Peta | Billiarde | Quadrillion |
|  |  |  |  |


| $10^{18}$ | Exa | Trillion | Quintillion |
| :--- | :--- | :--- | :--- |
| $10^{21}$ | Zetta | Trilliarde | Sextillion |

Beware! Confusion about billions or trillions is guaranteed, in particular because the British switched to the American way of counting. Of course the German way is the better way. Just look at your countries public debt. 5 Billion sounds just far friendlier than 5 Trillion.
The present American debt thus amounts to: 14 trillionus $\$$, 14 billionger $\$$, 14 Tera $\$\left(=14 \mathrm{~T} \$\right.$ ) or $1,4 \cdot 10^{13} \$$.

## Conversion from Enlightened to Benighted and Back

It remains to look at conversions from the enlightened decimal SI units to the outdated and benighted US units. Scientists like me and a lot of other smart people use the SI system because it is a hell of a lot easier than old nonmetric systems. Most people simply comply with the SI system (which was not always the case) because their governments long ago had signed the international treaty demanding the introduction of the SI system.
The Americans, however, prefer to do things the difficult way. They have staunch allies in this: Myanmar (formerly known as Burma) and Liberia. The rest of the world is metric.
Note that measuring temperature in degree Celsius or Fahrenheit is essentially a matter of taste. Not using a nonmetric or better non-decimal system for measuring length (and then automatically also area and volume) and weight is a matter of being stubborn or just plain stupid. Sorry, but that's the truth.
I wonder if you know one of the reason why the Americans are still hanging on to their illogical middle-age units (besides general stupidity related to the deplorable level of general education, and an unshakable believe that things as they are in the USA cannot possibly be improved upon). No? Then try this link. You will be amazed.
Now let's be a bit self-critical. All of us still use the ancient system for measuring time (and angles) that's not decimal but sexagesimal (base 60). It originated with the ancient Sumerians in the 3rd millennium BC, was passed down to the ancient Babylonians, and it is still used - in a modified form - for measuring time, angles, and the geographic coordinates in our modern times.
So we aren't completely metric either. But at least the ancient Sumerians had a system.
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There is no real system or any justification for inch, foot, yards, miles, acres, pints, gallons, ounces, pound, and so on, except that you can use it for a good one-liner:

## Most architects think by the inch, talk by the yard, and should be kicked by the foot

Prince Charles
(I tend to agree)

There is no easy way to convert from the SI metric system to the American non-metric non-system. You need to convert bit by bit. You could commit the rules to memory provided you have nothing better to do, like watching paint dry.

Use this table but be careful. I give no guarantee!

| USA (Red = major unit) |  |  |  | SI Metric |
| :---: | :---: | :---: | :---: | :---: |
| Length |  |  |  |  |
| 1 inch (in, sign ") | = | $1 \mathrm{in} ; 1^{\prime \prime}$ | = | 2,54 cm |
| 1 milli inch (mill) | = | 0,001 in | = | 0,0254 cm |
| 1 foot (ft, sign ' ) | = | 12 in | = | 0,3048 m |
| 1 yard (yd) | = | 3 ft | = | $\begin{gathered} 0,9144 \mathrm{~m} \\ \text { almost } 1 \mathrm{~m} \end{gathered}$ |
| 1 fathom | = | $2 \mathrm{yd}{ }^{1}$ | = | 1,8288 m |
| 1 rod | = | $5,5 \mathrm{yd}{ }^{1)}$ | = | 5,0292 m |
| 1 chain | = | $4 \operatorname{rod}{ }^{1)}$ | = | 20,1168 m |


| 1 furlong | $=$ | 10 chain ${ }^{1)}$ | $=$ | 201,168 m |
| :---: | :---: | :---: | :---: | :---: |
| 1 mile | $=$ | 80 furlong | = | 1609,344 m |
| 1 nautical mile | = | 1 n mile | = | 1852 m |
| Area |  |  |  |  |
| 1 sq.in. |  | 1 sq.in. | = | $\begin{gathered} (2,54 \mathrm{~cm})^{2} \\ 6,452 \mathrm{~cm}^{2} \end{gathered}$ |
| 1 sq.ft. | = | $\begin{aligned} & (12 \mathrm{in} .)^{2} \\ = & 144 \mathrm{sq} . \mathrm{in} . \end{aligned}$ | = | $929,1 \mathrm{~cm}^{2}$ |
| 11 sq.ft. |  | easy to convert | $\approx$ | $1 \mathrm{~m}^{2}$ |
| 1 acre | = | $\begin{aligned} & 4840 \text { sq.yd. } \\ = & 9 \cdot 4840 \text { sq.ft. } \\ = & 43.560 \text { sq.ft. } \end{aligned}$ | = | $0,405 \mathrm{ha}=4050 \mathrm{~m}^{2}$ |
| Liquid volumes |  |  |  |  |
| 1 US fluid ounce (fl oz) | = | 2 Tbsp (Tablespoon) or 6 Tsp (Teaspoon) or 8 US fluid drams ${ }^{2)}$ | = | $\begin{aligned} & 29,57 \mathrm{ml} \\ = & 29,57 \mathrm{~cm}^{3} \end{aligned}$ |
| 1 U.S. pint (pt) | = | 2 cups = 16 fl oz | = | 473.18 ml |
| 1 U.S. quart (qt) | $=$ | $2 \mathrm{pt}=32 \mathrm{fl} \mathrm{oz}$ | = | 0.951 |
| 1 U.S. gallon (gal) | = | $\begin{gathered} 4 \mathrm{qt}=128 \mathrm{fl} \mathrm{oz} \\ =231 \mathrm{cu} \mathrm{in} \end{gathered}$ | = | 3.781 |
| 1 barrel (bbl) | = | $\begin{gathered} 31,6 \text { ga } \\ =1 / 2 \text { hogshead } 1) \end{gathered}$ | = | 119.24 I |
| 1 oil barrel (also bbl) | = | $42 \mathrm{gal}=2 / 3$ hogshead | = | 1591 |
| 1 hogshead | = | $\begin{gathered} 63 \mathrm{gal} \\ =8.42 \mathrm{cu} \mathrm{ft} \\ =524.7 \mathrm{lb} \text { of water } \end{gathered}$ | = | 238.48 I |
| 1) I'm not making this up! <br> 2) I'm not making this up either and even spared you the Imperial fluid ounce $=0.960$ US fluid ounces and the US food labelling fluid ounce $=30 \mathrm{ml}=1.014420681$ US customary fluid ounces! |  |  |  |  |

