

## 4.2.2 Paramagnetism

The treatment of paramagnetism in the most simple way is exactly identical to the treatment of [orientation polarization](#). All you have to do is to replace the *electric dipoles* by magnetic dipoles, which we call *magnetic moments*.

- We have permanent dipole moments in the material, they have no or negligible interaction between them, and they are free to point in any direction *even in solids*.
- This is a major difference to electrical dipole moments which can *only* rotate if the whole atom or molecule rotates; i.e. only in liquids. This is why the treatment of magnetic materials focusses on ferromagnetic materials and why the underlying symmetry of the math is not so obvious in real materials.
- In an external magnetic field the magnetic dipole moments have a tendency to orient themselves into the field direction, but this tendency is opposed by the thermal energy, or better entropy of the system.

Using [exactly the same line of argument](#) as in the case of orientation polarization, we have for the potential energy  $W$  of a magnetic moment (or dipole)  $m$  in a magnetic field  $H$

$$W(\varphi) = - \mu_0 \cdot \underline{m} \cdot \underline{H} = - \mu_0 \cdot m \cdot H \cdot \cos \varphi$$

- With  $\varphi$  = angle between  $H$  and  $m$ .

In *thermal equilibrium*, the number of magnetic moments with the energy  $W$  will be  $N[W(\varphi)]$ , and that number is once more given by the Boltzmann factor:

$$N[W(\varphi)] = c \cdot \exp -(W/kT) = c \cdot \exp \frac{m \cdot \mu_0 \cdot H \cdot \cos \varphi}{kT} = N(\varphi)$$

- As before,  $c$  is some as yet undetermined constant.

As before, we have to take the component in field direction of all the moments having the same angle with  $H$  and integrate that over the unit sphere. The result for the induced magnetization  $m_{\text{ind}}$  and the total magnetization  $M$  is the same as before for the induced dielectric dipole moment:

$$m_{\text{ind}} = m \cdot \left( \coth \beta - \frac{1}{\beta} \right)$$

$$M = N \cdot m \cdot L(\beta)$$

$$\beta = \frac{\mu_0 \cdot m \cdot H}{kT}$$

- With  $L(\beta) = \text{Langevin function} = \coth \beta - 1/\beta$

The only interesting point is the *magnitude* of  $\beta$ . In the case of the orientation polarization [it was](#)  $\leq 1$  and we could use a simple approximation for the Langevin function.

- We know that  $m$  will be of the order of magnitude of [1 Bohr magneton](#). For a rather large magnetic field strength of  $5 \cdot 10^6 \text{ A/m}$ , we obtain as an estimate for an upper limit  $\beta = 1.4 \cdot 10^{-2}$ , meaning that the range of  $\beta$  is even smaller as in the case of the electrical dipoles.
- We are thus justified to use the [simple approximation](#)  $L(\beta) = \beta/3$  and obtain

$$M = N \cdot m \cdot (\beta/3) = \frac{N \cdot m^2 \cdot \mu_0 \cdot H}{3kT}$$

- The paramagnetic susceptibility  $\chi = M/H$ , finally, is

$$\chi_{\text{para}} = \frac{N \cdot m^2 \cdot \mu_0}{3kT}$$

Plugging in some typical numbers (A Bohr magneton for  $m$  and typical densities), we obtain  $\chi_{\text{para}} \approx +10^{-3}$ ; i.e. an *exceedingly small* effect, but with certain characteristics that will carry over to ferromagnetic materials:

- There is a strong temperature dependence and it follows the "**Curie law**":

$$\chi_{\text{para}} = \frac{\text{const}}{T}$$

- Since ferromagnets of all types turn into paramagnets above the Curie temperature  $T_C$ , we may simply expand Curie's law for this case to

$$\chi_{\text{ferro}}(T > T_C) = \frac{\text{const}^*}{T - T_C}$$

In summary, paramagnetism, stemming from some (small) average alignment up of permanent magnetic dipoles associated with the atoms of the material, *is of no (electro)technical consequence*. It is, however, important for analytical purposes called "**Electron spin resonance**" (**ESR**) techniques.

There are other types of paramagnetism, too. Most important is, e.g., the **paramagnetism of the free electron gas**. Here we have magnetic moments associated with spins of electrons, but in a *mobile* way - they are not fixed at the location of the atoms

- But as it turns out, other kinds of paramagnetism (or more precisely: calculations taking into account that magnetic moments of atoms can not assume any orientation but only some quantized ones) do not change the general picture: *Paramagnetism is a weak effect*.