4.1.2 Origin of Magnetic Dipoles

- Where are magnetic dipoles coming from? The classical answer is simple: A **magnetic moment *m*** is generated whenever a *current flows in closed circle*.
  - *Of course, we will not mix up the letter **m** used for magnetic moments with the m^*_e, the mass of an electron, which we also need in some magnetic equations*.
  - For a current I flowing in a circle enclosing an area A, m is defined to be

$$m = I \cdot A$$

  - This does not only apply to "regular" current flowing in a wire, but in the extreme also to a *single electron circling around an atom*.
- In the context of **Bohrs model** for an atom, the magnetic moment of such an electron is easily understood:
  - The current I carried by *one* electron orbiting the nucleus at the distance r with the frequency $\nu = \omega/2\pi$ is .

$$I = e \cdot \frac{\omega}{2\pi}$$

  - The area A is πr^2, so we have for the magnetic moment m_{orb} of the electron

$$m_{orb} = e \cdot \frac{\omega}{2\pi} \cdot \pi r^2 = \tfrac{1}{2} \cdot e \cdot \omega \cdot r^2$$

- Now the *mechanical* angular momentum L is given by

$$L = m^*_e \cdot \omega \cdot r^2$$

  - With m^*_e = mass of electron (*the * serves to distinguish the mass m^*_e from the magnetic moment m^e of the electron)*, and we have a simple relation between the mechanical angular momentum L of an electron (which, if you remember, was the decisive quantity in the Bohr atom model) and its magnetic moment m.

$$\underline{m}_{orb} = -\frac{e}{2m^*_e} \cdot \underline{L}$$

  - The *minus sign* takes into account that mechanical angular momentum and magnetic moment are antiparallel; as before we note that this is a *vector equation* because both m and L are (polar) vectors.
  - The quantity $e/2m^*_e$ is called the **gyromagnetic relation** or quotient; *it should be a fixed constant* relating m and any given L.
  - However, in real life it often deviates from the value given by the formula. How can that be?
  - Well, try to remember: Bohr's model is a mixture of classical physics and quantum physics and *far too simple* to account for everything. It is thus small wonder that conclusions based on this model will not be valid in all situations.
- In *proper quantum mechanics* (as in Bohr's semiclassical model) L comes in *discrete values only*. In particular, the fundamental assumption of Bohr's model was $L = n \cdot \hbar$, with n = quantum number = **1, 2, 3, 4, ...**
  - It follows that m_{orb} *must be quantized, too*; it must come in multiples of

$$m_{orb} = \frac{h \cdot e}{4\pi \cdot m^*_e} = m_{Bohr} = 9.27 \cdot 10^{-24}\ \mathrm{Am}^2$$

- This relation defines a **fundamental unit for magnetic dipole moments**, it has its own name and is called a **Bohr magneton**.
- It is for magnetism what an elementary charge is for electric effects.

But electrons orbiting around a nucleus are not the *only* source of magnetic moments.

- Electrons always have a **spin** s, which, on the level of the Bohr model, can be seen as a built-in angular momentum with the value $\hbar \cdot s$. The spin quantum number s is ½, and this allows two directions of angular momentum and magnetic moment , always symbolically written as .

$$s = \begin{cases} +1/2 \\ -1/2 \end{cases}$$

It is possible, of course, to compute the circular current represented by a charged ball spinning around its axis if the distribution of charge in the sphere (or on the sphere), is known, and thus to obtain the magnetic moment of the spinning ball.

- Maybe that even helps us to understand the internal structure of the electron, because we know its magnetic moment and now can try to find out what kind of size and internal charge distribution goes with that value. Many of the best physicists have tried to do exactly that.
- However, as it turns out, whatever assumptions you make about the internal structure of the electron that will give the right magnetic moment will always get you into *deep trouble* with *other properties* of the electron. There simply is *no internal structure* of the electron that will explain its properties!
- We thus are forced to simply accept as a *fundamental property of an electron* that it always carries a magnetic moment of

$$m^{e} = \frac{2 \cdot h \cdot e \cdot s}{4\pi \cdot m^{*}_{e}} = \pm m_{Bohr}$$

- The factor **2** is a puzzle of sorts - not only because it appears at all, but because it is actually **= 2.00231928**. But pondering this peculiar fact leads straight to quantum electrodynamics (and several Nobel prizes), so we will not go into this here.

The total magnetic moment of an atom - *still within the Bohr model* - now is given by the (vector)sum of all the "orbital" moments and the "spin" moments of all electrons in the atom, taking into account all the quantization rules; i.e. the requirement that the angular momentums L cannot point in arbitrary directions, but only in fixed ones.

This is were it gets complicated - even in the context of the simple Bohr model. A bit more to that can be found in the link. But there are few rules we can easily use:

- All *completely filled orbitals* carry *no* magnetic moment because for every electron with spin s there is a one with spin $-s$, and for every one going around "*clockwise*", one will circle "*counter clockwise*". This means:
- *Forget the inner orbitals* - everything cancels!
- Spins on not completely filled orbitals tend to *maximize their contribution*; they will first fill all available energy states with spin up, before they team up and cancel each other with respect to magnetic momentum.
- The *chemical environment*, i.e. bonds to other atoms, incorporation into a crystal, etc., may strongly change the magnetic moments of an atom.

The net effect for a given (isolated) atom is simple. Either it has a magnetic moment in the order of a Bohr magneton because not all contributions cancel - or it has none. And it is possible, (if not terribly easy), to calculate what will be the case. A first simple result emerges: Elements with an *even* number of electrons have generally *no magnetic moment*.

We will leave the rules for getting the permanent magnetic moment of a single atom from the interaction of spin moments and orbital moments to the more advanced (quantum theory) textbooks, here we are going to look at the possible effects if you:

- bring atoms together to form a solid, or
- subject solids to an external magnetic field H

A categorization will be given in the next paragraph.

Questionaire
Multiple Choice questions to 4.1.2