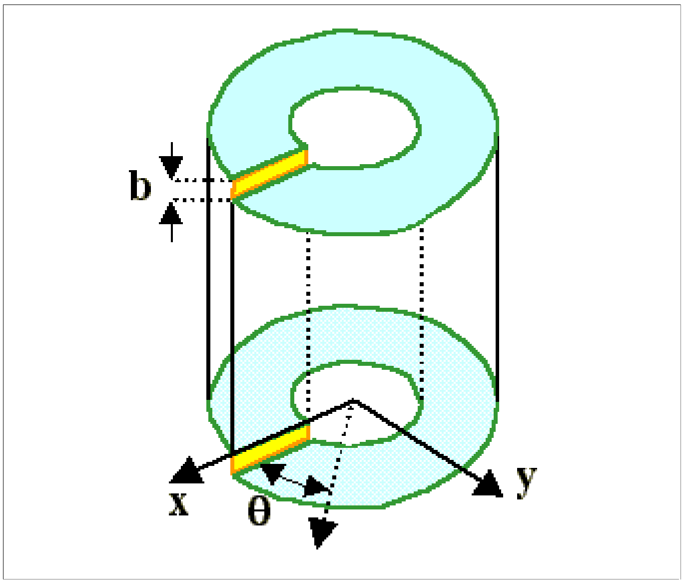
5.2.6 Essentials to 5.2: Dislocations - Elasticity Theory, Energy , and Forces

- Around a dislacotion is a *displacement field* (= vector field) , which defines a *strain field* (= tensor field), which gives cause to a *stress field* (= tensor field) via *elastic* relations. Stress times strain give the (potential) *energy* contained in these fields and thus the *energy of a dislocation*; derivatives of energy with respect to coordinates give *forces* acting on dislocations
  - The *displacement field* $\underline{u}(x,y,z)$ can be obtained by just looking hard at the dislocation - then write it down.
  - The rest is just Math - not all that easy, but not reyll difficult either.



$$u_z = \frac{b \cdot \theta}{2\pi} = \frac{b}{2\pi} \cdot \tan^{-1}(y/x) = \frac{b}{2\pi} \cdot \arctan(y/x)$$

- In cylinder coordinates (r, θ, z) rather simple expressions for the stress and the strain result but with the two major problems emerging as soon as we look at the energy *per unit length* of., e.g. a screw dislocation:

$$E_{el}(\text{screw}) = \frac{G \cdot b^2}{4\pi} \cdot \int_0^\infty \frac{dr}{r}$$

  - Both boundaries lead to infinte energy values!

- The first problem comes from overextending elastic theory, only good at small deformations, to the core region of the dislocation, the second one because the strain decreases so slowly that it is still felt far away form the dislcotion.
  - The problem gets repaired by defining an inner and outer cut-off radius r_o and R, respectively, adding some core energy E_{core}, worrying a lot if you are given to it, and finally coming out with an extermely simple, usually good enough, and very important approximation for the energy per length unit $|\underline{b}|$

$$E_{el} = \frac{G \cdot b^2}{4\pi} \cdot \int_{r_o}^{R} \frac{dr}{r} + E_{core} \approx \frac{G \cdot b^2}{4\pi(1-\nu)} \cdot \left(\ln \frac{e \cdot R}{b} \right)$$

$$E_{disl} \approx Gb^2$$

  - Putting numbers into the equation gives several **eV** per unit length $|\underline{b}|$ and thus tells us that dislocations tend to be straight lines (shortest possible length!).

A dislocation moves if forces are acting on it, causing plastic deformation. In other words: work $\boldsymbol{W = F \cdot A_S}$ is done if a dislocation sweeps over an area $\boldsymbol{A_S}$.

- The procedure for calculating the force is simple:

  - Take the forces ***F*** acting on your crystal.
  - Determine the component $\boldsymbol{F_g}$ in the glide plane of your dislocation that points in the direction of the Burgers vector ***b***.
  - Calculate the resolved shear stress $\tau_{\mathbf{res}}$ in the glide plane from the force component (= $\boldsymbol{F_g/A}$).
  - The force acting on a unit length of the disclotion is $\boldsymbol{F_{dis} = \tau \cdot b}$ and is always perpendicular to the line direction ***t***