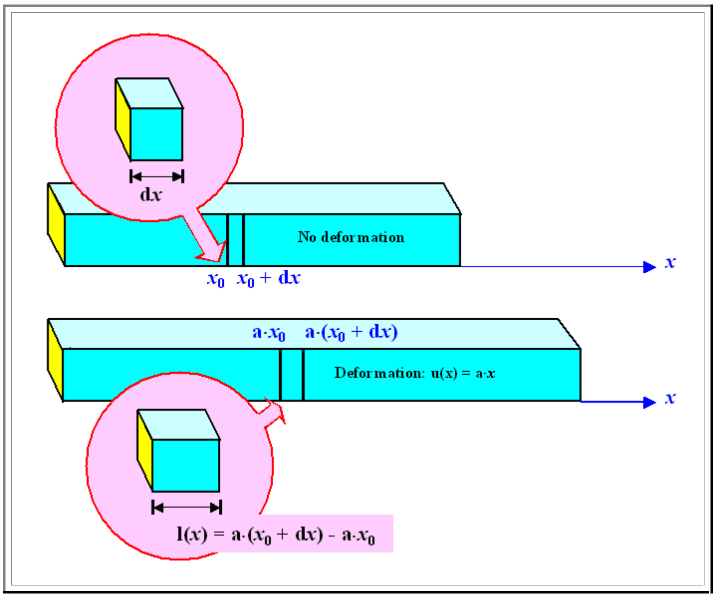
Displacement and Strain

Basics

- While the relation between the displacement field ***u*(*r*)** and the local strain tensor ϵ_{ij} is rather elementary, it does not hurt to recall the decisive points.
  - Let's take the simple example from the backbone and consider a rod that is uniformly elongated; i.e. $\underline{u}(\underline{r}) = u_x(x) = a \cdot x$; **a** is some constant.
  - In other words, the vector ***u*** *only* has a component in ***x***-direction, which *only* depends on ***x*** as variable. The geometry than looks like this:



  - At any point in the rod a little cube will be deformed into a ***cuboid*** - the side in ***x***-direction is somewhat longer than the others.
- What kind of strain do we have to put on a cube positioned a ***x***, to produce the cuboid?
  - Well, since there is only strain in ***x***-direction, we simply write down the elementary formula for strain

$$\epsilon_{xx} = \epsilon_x = \frac{l - l_0}{l_0} = \frac{u_x(x + dx) - u_x(x)}{dx} = \frac{du_x}{dx}$$

- If we deform in all three directions, we get corresponding expressions for ϵ_{yy} and ϵ_{zz}.
- Since we also might have displacement components in ***x***-direction that depend on ***y*** or ***z***, e.g. $u_x(x, y, z) = a \cdot y$, we may, in general, also form mixed (partial) derivatives; e.g. $\partial u_x(x, y, z)/\partial y$. What do those derivatives signify?
  - Shear stresses, of course. A little less easy to see, perhaps, but there can be no doubt about it.
  - You may want to try to show that for yourself with the simple displacement field given above and the equations in the backbone as a guideline for what you are looking for.