Stirlings Formula

Basics

- Stirlings formula is an indispensable tool for all combinatorial and statistical problems because it allows to deal with **factorials**, i.e. expressions based on the definition $\mathbf{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \ldots \cdot N := N!}$
- It exists in several modifications; all of which are approximations with different degrees of precision. It is relatively easy to deduce its more simple version. We have

$$\ln x! \;=\; \ln 1 + \ln 2 + \ln 3 + \ldots + \ln x \;=\; \sum_{1}^{x} \ln y$$

  - With **y =** positive integer running from **1 to x**
  - For large **y** we may replace the sum by an integration in a good approximation and obtain

$$\sum_{1}^{x} \ln y \;\approx\; \int_{1}^{x} (\ln y) \cdot dy$$

  - With $\int (\ln y) \cdot dy = y \cdot \ln y - y$, we obtain

$$\ln x! \;\approx\; x \cdot \ln x \;-\; x \;+ 1$$

- This is the simple version of Stirlings formula. it can be even more simplified for large ***x*** because then $x + 1 \ll x \cdot \ln x$; and the most simple version, perfectly sufficient for many cases, results:

$$\ln x! \;\approx\; x \cdot \ln x$$

- **However!!** We not only produced a simple approximation for ***x*!**, but turned a *discrete* function having values for integers only, into a *continuous* function, giving numbers for something like **3,141! -** which may or may not make sense.
  - This may have dire consequences. Using the Strirling formula you may, e.g., move from *absolute probabilities* (always a number between **0** and **1**) to *probability densities* (any positive number) without being aware of it.
- Finally, an even better approximation exists (the prove of which would take some **20** pages) and which is already rather good for small values of ***x***, say ***x* > 10**:

$$x! \;\approx\; (2\pi)^{1/2} \cdot x^{(x + ½)} \cdot e^{-x}$$