Illustration

Solution to Exercise 5.2-1: Fresnel Coefficients

- Consider an unpolarized light beam hitting the surface of an isotropic glass with **n = 1,5** at right angles (α = **0°**).
- Derive the simplified Fresnel equations as given in the script for α **= 0°**

- Setting the cosines to **1** in the **TE** or **TM** case gives

TE case	**TM case**
$E_{ref} = E_{in} \cdot \dfrac{\sin\beta - \sin\alpha}{\sin\beta + \sin\alpha}$	$E_{ref} = E_{in} \cdot \dfrac{\sin\beta - \sin\alpha}{\sin\beta + \sin\alpha}$
Division by sinβ; sinα/sinβ = *n*	
TE and TM	
$E_{ref} = -\, E_{in} \cdot \dfrac{n-1}{n+1}$ *q.e.d*	

- There is thus no difference between the **TE** and **TM** case.

- How much of the light will be reflected?

- We can use the simple equations for perpendicular incidence from above, re-written for intensities:

$$\frac{I_{ref}}{I_{in}} = \left(\frac{n-1}{n+1}\right)^2$$

- The intensity of the reflected light is thus

$$I_{ref} = I_{in}\left(\frac{n-1}{n+1}\right)^2 = I_{in}\left(\frac{0{,}5}{2{,}5}\right)^2 = 0{,}04\, I_{in}$$

- The reflected intensity is thus **4 %** of the incoming intensity.

- What is the phase relation between incoming, reflected and transmitted light?

- To answer that question we must look at the field strength. There is a *minus sign* and the the phase of the reflected beam thus is phase-shifted by **180 ° =** π

- How does the beam leave the crystal (Intensity and polarization)?

- For the transmitted beam we have the simple relation $I_{tr} = I_{in} - I_{re}$. The intensity of transmitted beam thus is **96 %** of the incoming intensity for an optical material with ***n* = 1,5**.
  - At the "exit" from the ***n* = 1,5** optically dense material to the ***n* = 1** less dense material α and β interchange their role or **sinβ/sinα = *n* = 1,5** now. Taking this into account we get the same equations for the perpendicular incidence as above but without the *"–" (minus) sign* for the field strengths. This means that **96 % of 96 % (= 92,16 %)** of the incoming beam exits the optical material.
  - How about phases? Looking at the original Fresnel equations for the electrical field strength we see that in both the **TE** and **TM** case there is no sign and therefore phase jump for the *transmitted* wave for all angles (since α, β **<= 90 ^{o}** in all cases). For the reflected wave at the optically less dense medium (the wave reflected back into the interior of the optical material) there isn't a phase change either because the minus sign is no longer there.

- Now consider these questions for some polarization of the incoming light.

- No difference since we have the same equations for both the **TE** and **TM** case.