Solution to Exercise 5.1-3 Polarization

Illustration

1. question:: A light beam with intensity I_0 passes through *one* ideal polarizer.

1. How does the intensity relate to the electrical field strength?

Intensity neasures the energy or better *power flux* contained in the light. it is propotional to the square of the electrical field strength

2. The incoming ("input") light beam is unpolarized. How large is the intensity at the output?

It's obviously $I_0/2$. Considering this picture we can decompose the light beam in two fully polarized beams, each having the intensity $I_0/2$. The polarizer takes out one of the beams and $I_0/2$ remains.

3. Does this intensity change if you rotate the polarizer around the axis coinciding with the propagation direction of the light = optical axis?

No.

4. The incoming light beam is **100 %** linearly polarized. How large is the intensity on the output as a function of the angle between polarization direction of the light and polarizing direction of the polarizer.

The intensity must vary betwen **100 %** and **0 %** of the incoming intensity for an angle of $\alpha = 0^o$ and 90^o , respectively.
For an arbitrary angle α we have a field strength $E(\alpha) = E_0 \cdot \cos\alpha$; the transmitted intensity then scales with $(\cos\alpha)^2$ between the extremes.

2. question:A light beam with intensity I_0 first passes through one ideal polarizer, and then through a *second* one. Both polarizers can be rotated freely around the optical axis.

1. The light beam is unpolarized. How large is the intensity on the output if both ideal polarizers are in parallel?

It's still $I_0/2$ because two polarizers in parallel behave just like one.

2. The light beam is unpolarized. How large is the intensity on the output if the ideal polarizers are "crossed", i.e. their polarization directions are at right angles?

The intensity is zero.

3. The light beam is **100 %** linearly polarized. How large is the intensity of the output as a function of the variable angle α between the two polarizing directions of the polarizers and the fixed angle β between the polarization direction of the light and the first polarizer it encounters? Note that in this case you rotate the *second* polarizer.

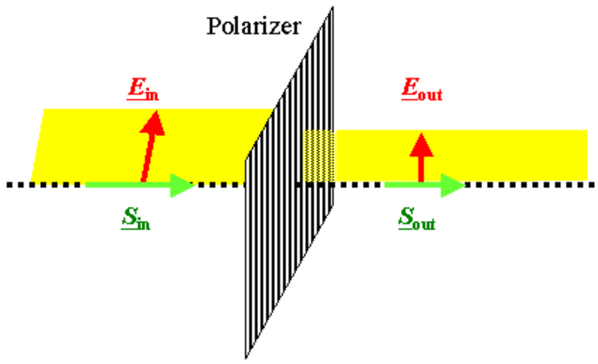
After passing through the first polarizer, the intensity is $I_0 \cdot (\cos\beta)^2$. After passing through the second polarizer we have $I = I_0 \cdot (\cos\beta)^2 \cdot (\cos\alpha)^2$

4. Does the result for the question above change if you rotate the *first* polarizer and keep the second one at the fixed angle β?

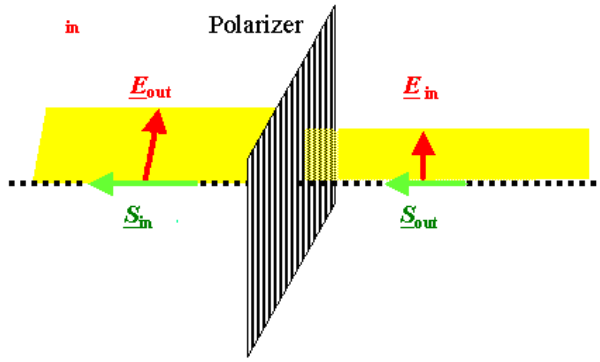
No.

5. Is for all of the above the direction of the light paths always reversible as stated before?

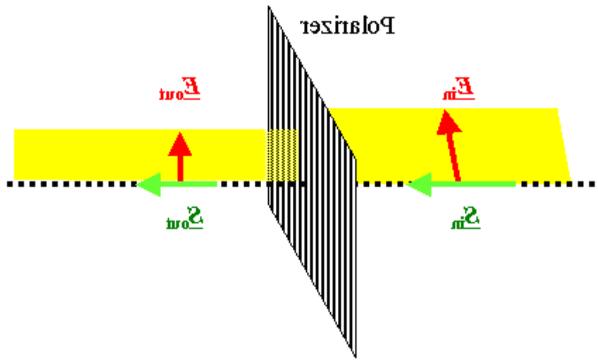
Tricky. Just reversing arrowheads obviously doesn't work.
You can't get a higher intensity from a lower one as you would if you just reverse the sign of the Poyntig vector as shown below.
Using a mirror symmetry works but this trival.



Starting situation.



Reversing vector signs does not work.



Mirroring works.

3. question: Now consider a system with *two* fixed *crossed* polarizers and a *third* one that can be rotated *in between* the two crossed ones.

1. The incoming light beam is unpolarized. How large is the intensity of the output as a function of the variable angle α between the first (fixed) and the third polarizer that can be rotated?

After the first polarizer, the intensity is $I_0/2$; it decreases with $\mathbf{(\cos\alpha)^2}$ behind the third polarizer that can be rotated . The second (fixed and crossed with respect to the first one) polarizer transmits components with a $\mathbf{(\sin\alpha)^2}$ dependence (make a simple drawing and do the geometry!) so all together we have for the output

$I(\alpha) = \dfrac{I_0}{2} \cdot (\cos\alpha)^2 \cdot (\sin\alpha)^2$	$I(\alpha = 0^o) = 0$
	$I(\alpha = 45^o) =$ **maximal value** $= (I_0/2) \cdot 0{,}25$
	$I(\alpha = 90^o) = 0$

2. The incoming light beam is **100 %** linearly polarized. How large is the intensity on the output as a function of the variable angle α between the first (fixed) and the third polarizer (can be rotated) considering that the angle β between the incoming light polarization and the polarization direction of the first polarizer is fixed at a value β?

As above except that the intensity after the first polarizer is now $I_0 \cdot (\cos\beta)^2$