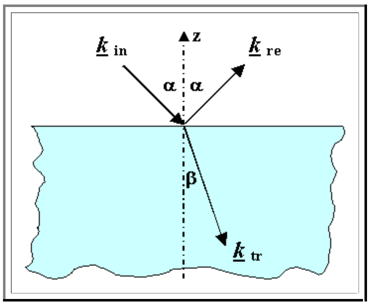
Solution to Exercise 5.1-1: Derivation of Snellius Law

Illustration

Show that you obtain $I_{tr} = I_{in} - I_{ref}$ and Snellius law (**sinα/sinβ = *n***) from energy and momentum conservation



Solution:

- The intensity I of the beams is given by their power (energy /t) which is given by the nunber of photons/s in the beams: E_{in}, E_{re}, E_{tr}. Everythin always per cm^2 but that is not important for what follows.
- *Energy conservation* demands

$$E_{in} = E_{re} + E_{tr}$$

$$E_{tr} = \; = E_{in} - E_{re}$$

$$I_{tr} = \; I_{in} - I_{re}$$

Looking at the **x**-component of the momentum ***p*** and considering that the wavelength in the material is λ/***n*** we have

$$|p_{z,\,in}| = I_{in}\hbar k_{in} \cdot \sin\alpha = \frac{I_{in}\hbar \cdot 2\pi \cdot \sin\alpha}{\lambda}$$

$$|p_{z,\,re}| = I_{re}\hbar k_{re} \cdot \sin\alpha = \frac{I_{re}\hbar \cdot 2\pi \cdot \sin\alpha}{\lambda}$$

$$|p_{z,\,tr}| \quad I_{tr}\hbar k_{tr} \cdot \sin\beta = \frac{I_{tr}\hbar \cdot 2\pi \cdot \sin\beta \cdot n}{\lambda}$$

- *Momentum conservation* demands that $p_{z,\,in} + p_{z,\,tr} - p_{z,\,re} = 0$, or

$$I_{in}\sin\alpha + I_{tr}\sin\beta \cdot n - I_{re}\sin\alpha = 0$$

- Substituting $I_{re} = I_{in} - I_{tr}$ leads straight ot

$$n = \frac{\sin\alpha}{\sin\beta}$$