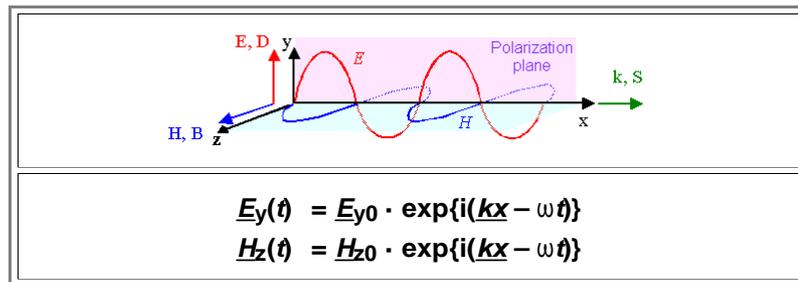## 5.1.4 Energy Flow, Poynting Vector and Polarization

Energy Flow and Poynting Vector

Light as a wave, by definition, is *polarized* since an arbitrary light wave can always be decomposed into plane waves described by the electrical field vector $\underline{E}(\underline{r}, t) = \underline{E}_0 \exp\{i(\underline{kr} - \omega t)\}$ and the corresponding expression for the magnetic field.

The electrical field *vector* (or the electrical displacement vector $\underline{D}$ in material) thus is contained in some plane (the **polarization plane**) that contains the direction of propagation (direction of $\underline{k}$-vector). Looking for example at a picture we had before but now augmented with the relevant *vector quantities*, we see that in this case:

- Polarization plane=**(y,x)** plane.
- Propagation direction=$\underline{k}$ direction=*Poynting vector $\underline{S}$* direction=**x** direction here.
- *E*-field amplitude in **y**-direction given by E_0.
- *H*-field amplitude in **z**-direction given by H_0.

$$\underline{E}_y(t) = \underline{E}_{y0} \cdot \exp\{i(\underline{kx} - \omega t)\}$$
$$\underline{H}_z(t) = \underline{H}_{z0} \cdot \exp\{i(\underline{kx} - \omega t)\}$$

This all looks well-known and clear. Usually we don't bother to go much deeper at this point. In a Master study course, however, you now ponder the picture and the equations above for a while. Then a few questions should come up, for example:

- Can polarization be described by a *vector*? Could I just use a unit vector in E_0-direction?
- What happens if E_0; H_0 are *not* parallel to D_0; B_0, something to be expected in anisotropic materials?
- How do I describe polarization in the *particle picture*?
- Is the electrical and magnetic field really *in phase* as shown above?
- How *large*, in **V/cm**, is the electrical field E_0 in a typical light wave. How large (roughly) is it for *one* photon of a given energy?
- Let's assume you know the electrical field E_0: how large is the *magnetic field H_0* associated with it?
- The energy a photon brings along is $E_{Ph}=h\nu$. How is this *light energy* transported by an electromagnetic wave?

It's not very difficult to conceive and to understand these questions. How about the answers?

The bad news is: There are no easy answers for some of those questions.

The good news is: For most of what follows you don't have to worry about the answers. I will go into details later whenever necessary. For now you can use this link to get some ideas about the answers and, even better, you could do the simple but illuminating exercise *below*.

Let's start with the last question. Light flow equals energy flow. All of us who once experienced a sun burn (or shot down one of those evil alien space ships with a Laser gun) know that. How does light energy "flow" exactly?

Well, we (should) know how much energy *density* **W =energy /cm³** is contained in an electric or magnetic field in vacuum (in materials use **D** and **B** instead of **E** and **H**): We have:

$$W_{elect} = \frac{\epsilon_0 \cdot E^2}{2} \qquad W_{mag} = \frac{\mu_0 \cdot H^2}{2}$$
$$[W_{elect; \; magn}] = [Ws \; m^{-3}]$$

We do not yet know, how **E** and **H** are linked. All we need to know is that this follows "straight" from the Maxwell equations and is given by

$$E_0 = \left(\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}\right)^{1/2} \cdot H_0 = Z_w \cdot H_0$$

● We defined a new quantity $Z_w = E_0/H_0 = (\mu_0\mu_r/\epsilon_0\epsilon_r)^{1/2}$ that is called the **wave impedance** of the medium. This is an apt name since Z_w does have the dimension "*Ohm*" (Ω) as required for an impedance.

● For the **vacuum impedance** (ϵ_r, μ_r=1) we get

$$Z_w(\text{Vacuum}) = \textbf{376,7 } \Omega$$

● This is a bit strange if you think about it. We just connected vacuum, the *nothing*, with a defined property that can be expressed in a simple number! You can buy a resistor with **377** Ω and wonder why it somehow reflects a property of vacuum.

▮ Be that as it may, we now define a *new* vector **S** as follows:

$$\underline{S} = \underline{E} \times \underline{H}$$

$$= \textbf{Poynting vector}$$

● The *Poynting vector* has interesting properties:

- **S** points in the direction of the propagation of the light, i.e. in wavevector **k** direction as shown in the figure above.
- $|\underline{S}| = S = E_0 \cdot H_0 \cdot \cos^2(\underline{k}\underline{r} - \omega t)$, the magnitude of the Poynting vector, gives directly the **power flux** in **W/cm^2** of an electromagnetic wave
- The average power delivered to an area is thus .

$$\langle S \rangle = \frac{E_0 H_0}{2} = \frac{E_0^2}{Z_w} = E_0^2 \cdot \left(\frac{\epsilon_r\epsilon_0}{\mu_r\mu_0}\right)^{1/2}$$

$$\langle S \rangle \propto E_0^2 \cdot \epsilon_r^{1/2} \propto E_0^2 \cdot n$$

● Don't mix up the **power flux** in **W/cm^2** with the **specific energy** measured in in **Ws/cm^3** or **J/cm^3** that is contained in fields as shown above! It's time for an exercise to get some ideas about the *numbers* associated with light power and field strength!

Exercise 5.1.2
Energy, field strength and photons

▮ We still have a bunch of open questions from above but before we tackle those we have to learn a bit about polarization.

Polarization of Light

▮ It is clear that an idealized *plane* wave, given essentially by the **exp[i($\underline{k}\underline{r}$ – ωt)]** term, describes a *monochromatic* and fully *coherent* light wave propagating in exactly *one direction* that is also **linearly polarized** by definition.

● Some Laser beams may come close to emitting a plane wave matching that description but laser light is a far cry from "normal" light.

● Most real or normal light beams, e.g. the light "beams" emanating from light bulbs, are *not polarized* as we know. "Real" light from light bulb looks - very schematically - as shown below. We are forced to assume that the electromagnetic wave has a beginning in space and time (at the light bulb, like now) and an end. The length of the whole thing we call "**coherence length**". The color (=wavelength), and the polarization plane for one of those waves with a beginning and an end, has some fixed value but that is different from the next wave coming along. The distribution of wavelengths we call the *spectrum* of the light source, and the distribution of polarization planes will be random. Each ones of theses "wavelets" is caused by an electron changing its state from higher to lower energy.

Coherence lenght

Spherical "Plane" wave
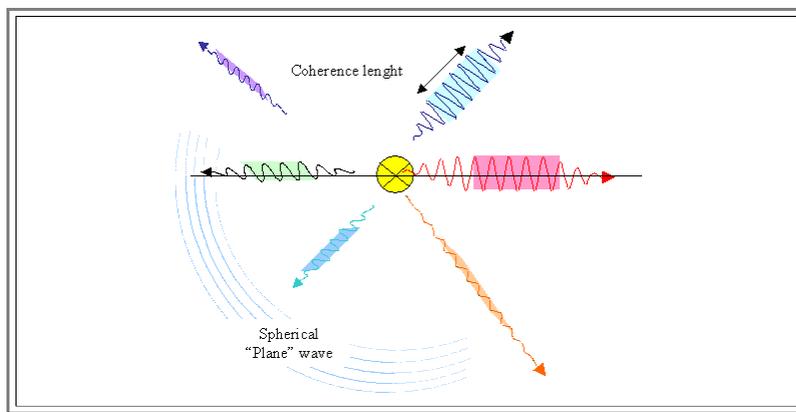
● The picture might be interpreted as *photons* coming out of the light bulb. If one of those wavelets would be seen as a completely *abstract symbol* for a photon, this would be true. However. photons are *not* just short pieces of an electromagnetic wave, so one must not interpret this figure as a *picture* of photons.

▶ We have a number of *new* questions to consider (never mind that we did not answer all the old ones yet):

1. How do we *detect* if and in which direction a light beam is polarized?
2. How can we *polarize* an unpolarized light beam if the need arises?
3. Does is matter if and how a light beam is polarized for the *applications* we have in mind?
4. If it matters: how do we *optimize* the needed polarization?
5. How does the polarization of a wave relate to *photon properties*?
6. Are there other *polarization modes* besides *linear* polarization?

We are now at the point where serious optics starts. It ain't exactly easy but we will only scratch the surface here.

▶ Let's look at these questions one by one. We will encounter rather difficult topics that we only treat in a *perfunctory* way here. Some topics will come up later again.

▶ **1. How do we *detect* if and how a light beam is polarized?**

● By using a **polarizer**, i.e. some material that transmits *only* that **E**-field component of an incoming light beam that is parallel to some material specific **polarization direction** as shown below.
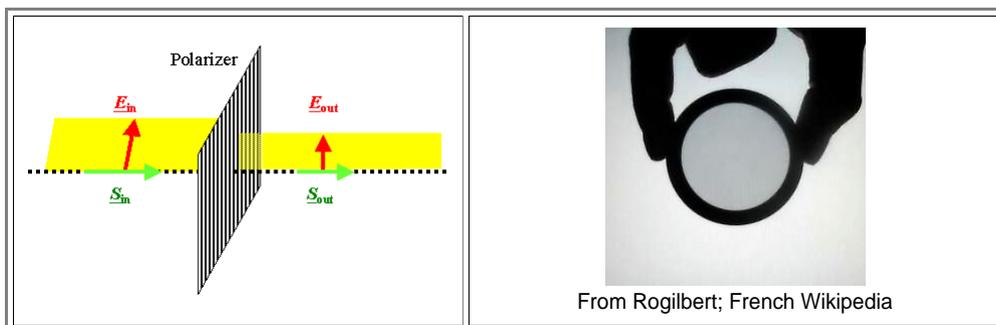

From Rogilbert; French Wikipedia

● Turn your polarizer and the light intensity coming out stays either constant if the incoming beam is not polarized at all (but is always lower in intensity) or varies between zero and maximum (=incoming) intensity in a **cos²** fashion if the incoming beam is **100 %** linearly polarized, as shown above. If the beam is *partially* polarized you have a mix of the two extremes.

▶ The second question (**2. How can we *polarize* an unpolarized light beam if the need arises?**) is answered too: Use a *polarizer*! Of course we have a new question now, the really tough one.

● **7. What, *exactly*, gives a material polarizing properties?**
And how can I manipulate or engineer those properties? That's indeed a tough question as we shall see in one of the next sub chapters. Before we try to answer this one, we look at a few simple things first.

What we perceive now is that a standard non-polarized light beam simply consists of many intrinsically polarized beams but with randomly distributed polarization directions. The total polarization effect then is zero.
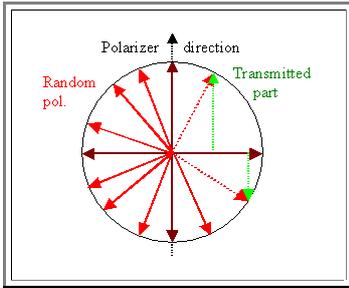
- This is easy to perceive with the help of the figure on the left.
- Randomly distributed *polarization vectors* sum up to zero=no polarization.
- Note that we now indirectly answered one of the earlier [questions from above](): Yes, you can use a **polarization vector** like we do in this case and most others. But beware! Polarization in general is more tricky than you might think and a simple vector is not always enough to describe it mathematically. (Remember the problems encountered in using a polar vector for describing rotations?). You will need a matrix if you go about it systematically.
- The picture also makes clear why the polarized beam emerging from a polarizer has a lower intensity than the incoming beam, and also how much lower it will be.
- If we want to make life easier we can replace a large number of beams with random polarization directions (red vectors) by just two beams at right angles (brown vectors).
- It is time for a quick exercise:

Exercise 5.1.3
Polarization

3. Does is matter if and how a light beam is polarized for the *applications* we have in mind?

- The answer is simple: No, it does not matter for many *simple* or *standard* applications. Your glasses, binoculars, microscopes, and so on, work perfectly well with "normal" light. With a polarization filter your camera will make somewhat better pictures in certain circumstances but that is usually not decisive.
- However, for anything a bit more sophisticated, like all the optical things that a *Material Scientist and Engineer* needs for her or his work or develops for others, it will matter *very much* as we will see. Here is a table that should give you some idea:

Application	Polarization matters *a lot*	Polarization doesn't matter *much*
Simple optical instruments (Cameras, binoculars, ...)		Forget it
Better optical instruments (Cameras, ..	you use a "Pol filter"	
Optical communications	not yet but soon	most cases now
Optical measurements	most depend on polarization	
3-dim movies	including circular polarization	
Laser	in many cases	some cases
LCD displays	impossible without polarization	

4. If it matters: how do we *optimize* the needed polarization?

- The answer is simple again: by constantly coming up with new or improved *materials*. Note the extremely simple and important truth:

5. How does the polarization of a wave relate to *photon* properties?

- Well - the photon has a spin of **1/2**! Naively put, there is a kind of (polar) vector associated with it. So there might be a relation between polarization and spin?

- Yes - up to a point. But it's not that simple! Whatever the case may be, my advice is: *forget it*! Whenever polarization comes up, *only* think in the wave picture. Otherwise you simply run into all kinds of unnecessary problems. However, it is perfectly possible to describe all the effects of polarization in the particle picture, too. There is no contradiction.

We have the last questions to consider and that will open a large can of worms, as the saying goes. Let's start with No. 6: Are there other *polarization modes* besides *linear* polarization?

- Yes there are. They are called *circular* and *elliptical* polarization. Here we go:

Other Modes of Polarization

Some idealized plane wave travelling in *z* direction with the electrical field vector pointing in *x*-direction writes as $E_x(z,t) = E_{0,x} \exp\{i(kz - \omega t)\}$ for the the electric field part. This idealized light wave is a solution of the Maxwell equations for some (idealized) conditions.

$E_y(z,t) = E_{0,y} \exp\{i(kz - \omega t + \pi/2)\}$; i.e. the same wave but with the electrical field pointing in *y*-direction *and a phase difference* of π/**2** (or any other you care to insert) is also a solution, of course.

If those waves travel inside a material, all we have to do is to replace the electrical field **_E_** by **_D_**, the dielectric displacement (and **_H_** by **_B_**, of course), and to take care of the change in wavelength or the magnitude of **_k_**, respectively.

- If the material is *isotropic* and *linear*, we have $\underline{D} = \epsilon_0 \epsilon_r \underline{E}$. That means that not only the two waves from above are still a solution of the Maxwell equations for the case considered, *so is any superposition* of those (and possibly other) solutions. So whatever we are about to discuss works just as well in "simple" materials. Most optical materials are amorphous (called "glass"), and meet the requirement *isotropic* and *linear* in a fair approximation so we don't have to worry *yet*.

- However, many crystalline materials are neither *isotropic* nor *linear* - and then we will be in trouble. Those materials are of course very interesting to a Materials Scientist but the theory of what happens is difficult and we will deal with that later.

Now let's add up the two **_E_** (or **_D_**) solutions from above and see what we get.

$$E_x(z,t) + E_y(z,t) = E_{0,x} \exp\{i(kz - \omega t)\} + E_{0,y} \exp\{i(kz - \omega t + \pi/2)\}$$

The real part of this complex equation gives the electrical field vector, we have

$$\underline{E} = E_{0,x} \cdot \cos(kz - \omega t) + E_{0,y} \cdot \sin(kz - \omega t)$$

- The full electrical field **_E_** thus is obtained by adding two vectors at right angles with magnitudes that change sinusoidally between ($\pm \underline{E}_0$). The total effect is a vector with length **_E_$_0$** that *rotates* with a circle frequency ω around the **_z_**-axis.

- What you get is called **circular polarization** for obvious reasons; it looks like shown below. Note that all of this appears to be far more complicated than it actually is, this is made clear by the animation below which shows the basic simplicity of what is going on.

(From wikipedia)
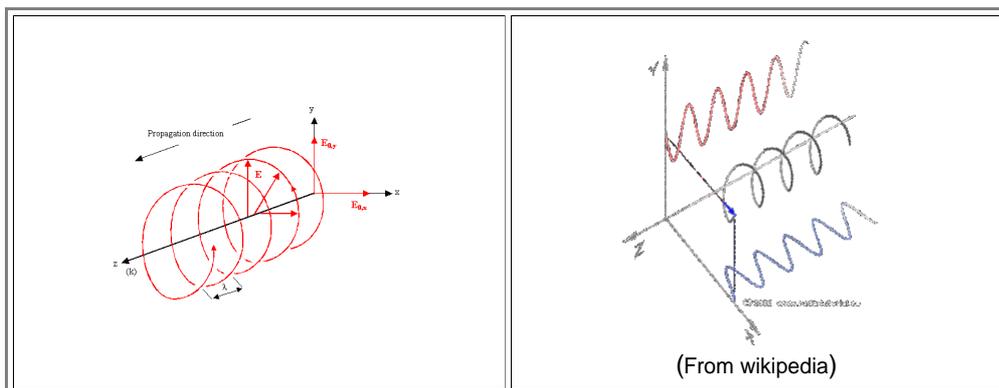
You realize, if you think about it, that we just opened the promised rather large can of worms. We started with simple, intuitively clear linear polarization and now progressed to circular polarization by superimposing two *special* linearly polarized waves. Of course, we can produce two basic kinds of circular polarization: **left-handed** and **right-handed** circular polarized light this way. Equally of course, we can now superimpose *all kinds of waves* with all kinds of phases. *Where will it end*?

- Actually, it's not as bad as it might be. Superimposing whatever you like just gives you **elliptically polarized light** (plus, perhaps, a background of unpolarized light).

- The extremes of an ellipse are straight lines (*linearly* polarized light, just superimpose the two waves without a phase difference) and the circle (phase difference $\pi/2$; equal amplitudes).

We will stop at this point. Far more important than going into intricate details of all this polarization stuff is to consider:

- Question 2 from above, not yet addressed: How do we *make* all these kinds of polarization? (*Answer*: with special *materials*, of course).
- What is it good for? Who needs circular or elliptically polarized light? (*Answer*: you - if you watch, e.g., **3D**-movies)

- We will look very briefly at some answers in the next chapter, where we deal with *light and materials*.

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Questionaire

Multiple Choice questions to 5.1.4

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