

Solution to Exercise 2.1-2

Derive numbers for v_0 , v_D , τ , and l

First Task: Derive a number for v_0 (at room temperature). We have

$$v_0 = \left(\frac{3kT}{m} \right)^{1/2} = \left(\frac{8,6 \cdot 10^{-5} \cdot 300 \text{ eV} \cdot \text{K}}{9,1 \cdot 10^{-31} \text{ K} \cdot \text{kg}} \right)^{1/2} = 1,68 \cdot 10^{14} \cdot \left(\frac{\text{eV}}{\text{kg}} \right)^{1/2}$$

- The dimension "square root of eV/kg " does not look so good - for a velocity we would like to have m/s . In looking at the energies we equated kinetic energy with the classical dimension $[\text{kg} \cdot \text{m}^2/\text{s}^2] = [\text{J}]$ with thermal energy kT expressed in $[\text{eV}]$. So let's convert eV to J (use the [link](#)) and see if that solves the problem. We have $1 \text{ eV} = 1,6 \cdot 10^{-19} \text{ J} = 1,6 \cdot 10^{-19} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$ which gives us

$$v_0 = 1,68 \cdot 10^{14} \cdot \left(\frac{1,6 \cdot 10^{-19} \text{ kg} \cdot \text{m}^2}{\text{kg} \cdot \text{s}^2} \right)^{1/2} = 5,31 \cdot 10^4 \text{ m/s} = 1,91 \cdot 10^5 \text{ km/hr}$$

Possibly a bit surprising - those electrons are no sluggards but move around rather fast. Anyway, we have shown that a value of $\approx 10^4 \text{ m/s}$ [as postulated in the backbone](#) is really OK.

- Of course, for $T \rightarrow 0$, we would have $v_0 \rightarrow 0$ - which should worry us a bit ????. If instead of room temperature ($T = 300 \text{ K}$) we would go to let's say 1200 K , we would just double the average speed of the electrons.

Second Task: Derive a number for τ . We have

$$\tau = \frac{\sigma \cdot m}{n \cdot e^2}$$

First we need some number for the concentration of free electrons per m^3 . For that we complete the [table given](#), noting that for the number of atoms per m^3 we have to divide the density by the atomic weight.

Atom	Density [$\text{kg} \cdot \text{m}^{-3}$]	Atomic weight $\times 1,66 \cdot 10^{-27} \text{ kg}$	Conductivity σ $\times 10^5 [\Omega^{-1} \cdot \text{m}^{-1}]$	No. Atoms [m^{-3}] $\times 10^{28}$
Na	970	23	2,4	2,54
Cu	8.920	64	5,9	8,40
Au	19.300	197	4,5	5,90

- So let's take $5 \cdot 10^{28} \text{ m}^{-3}$ as a good order of magnitude guess for the number of atoms in a m^3 , and for a first estimate some average value $\sigma = 5 \cdot 10^5 [\Omega^{-1} \cdot \text{m}^{-1}]$. We obtain

$$\tau = \frac{5 \cdot 10^5 \cdot 9,1 \cdot 10^{-31} \text{ kg} \cdot \text{m}^3}{5 \cdot 10^{28} \cdot (1,6 \cdot 10^{-19})^2 \Omega \cdot \text{m} \cdot \text{A}^2 \cdot \text{s}^2} = 3,55 \cdot 10^{-16} \frac{\text{kg} \cdot \text{m}^2}{\text{V} \cdot \text{A} \cdot \text{s}^2}$$

Well, somehow the whole thing would look much better with the unit $[\text{s}]$. So let's see if we can remedy the situation.

- Easy: Volts times Amperes equals *Watts* which is power, e.g. energy per time, with the unit $[\text{J} \cdot \text{s}^{-1}] = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$. Insertion yields

$$\tau = 1,42 \cdot 10^{-28} \frac{\text{kg} \cdot \text{m}^2 \cdot \text{s}^3}{\text{kg} \cdot \text{m}^2 \cdot \text{s}^2} = 3,55 \cdot 10^{-16} \text{ s} = 0,35 \text{ fs}$$

The backbone thus is right again. The scattering time is in the order of [femtosecond](#) which is a short time indeed. Since all variables enter the equation linearly, looking at somewhat other carrier densities (e.g. more than 1 electron per atom) or conductivities does not really change the general picture very much.

Third Task: Derive a number for v_D . We have (for a field strength $E = 100 \text{ V/m} = 1 \text{ V/cm}$)

$$\begin{aligned} |v_D| &= \frac{E \cdot e \cdot \tau}{m} = \frac{100 \cdot 1,6 \cdot 10^{-19} \cdot 3,55 \cdot 10^{-16}}{9,1 \cdot 10^{-31}} \frac{\text{V} \cdot \text{C} \cdot \text{s}}{\text{m} \cdot \text{kg}} = 6,24 \cdot 10^{-3} \frac{\text{V} \cdot \text{A} \cdot \text{s}^2}{\text{m} \cdot \text{kg}} \\ &= 6,24 \cdot 10^{-3} \frac{\text{kg} \cdot \text{m}^2 \cdot \text{s}^2}{\text{m} \cdot \text{kg} \cdot \text{s}^3} = 6,24 \cdot 10^{-3} \text{ m/s} = 6,24 \text{ mm/s} \end{aligned}$$

This is somewhat larger than the [value given in the backbone text](#).

- However - a field strength of **1 V/cm** applied to a *metal* is huge! Think about the current density j you would get if you apply **1 V** to a piece of metal **1 cm** thick.
- It is actually $j = \sigma \cdot E = 5 \cdot 10^7 [\Omega^{-1} \cdot \text{m}^{-1}] \cdot 100 \text{ V/m} = 5 \cdot 10^9 \text{ A/m}^2 = 5 \cdot 10^5 \text{ A/cm}^2$!
- For a more "reasonable" current density of **10^3 A/cm^2** we have to reduce E hundredfold and then end up with $|v_D| = \mathbf{0,0624 \text{ mm/s}}$ - and that is slow indeed!

Fourth Task: Derive a number for l . We have

$$l_{\min} = 2 \cdot v_0 \cdot \tau = 2 \cdot 5,31 \cdot 10^4 \cdot 3,55 \cdot 10^{-16} \text{ m} = 3,77 \cdot 10^7 \text{ m} = 0,0377 \text{ nm}$$

- Right again! If we add the comparatively miniscule v_D , nothing would change. Decreasing the temperature would lower l to eventually zero, or more precisely, to $2 \cdot v_D \cdot \tau$ and thus to a value far smaller than an atom..