2.6 Summary: Conductors

- What counts are the *specific* quantities:
  - Conductivity σ (or the specific resistivity $\rho = 1/\sigma$)
  - current density $\underline{j}$
  - (Electrical) field strength · $\underline{E}$

[σ] = (Ωm)$^{-1}$ = S/m; **S = 1/** Ω = "Siemens"
[ρ] = Ωm

- The basic equation for σ is:
  n **=** concentration of carriers
  μ = mobility of carriers

$$\sigma = |q| \cdot n \cdot \mu$$

- Ohm's law states:
  It is valid for metals, but not for all materials

$$\underline{j} = \sigma \cdot \underline{E}$$

- σ (of conductors / metals) obeys (more or less) several rules; all understandable by looking at ***n*** and particularly **μ**.
  - Matthiesen rule
    Reason: Scattering of electrons at defects (including phonons) decreases **μ**.

$$\rho = \rho_{\mathrm{Lattice}}(T) + \rho_{\mathrm{defect}}(N)$$

  - "**ρ(*T*)** rule":
    about **0,04 %** increase in resistivity per **K**
    Reason: Scattering of electrons at phonons decreases **μ**

$$\Delta\rho = \alpha_\rho \cdot \rho \cdot \Delta T \approx \frac{0{,}4\%}{^\circ\mathrm{C}}$$

  - Nordheim's rule:
    Reason: Scattering of electrons at **B** atoms decreases **μ**

$$\rho \approx \rho_A + \mathrm{const.} \cdot [B]$$

- Major consequence: You can't beat the conductivity of pure **Ag** by "tricks" like alloying or by using other materials. (Not considering superconductors).

- Non-metallic conductors are *extremely* important.
  - Transparent conductors (TCO's)
    ("ITO", typically oxides)

    No flat panels displays = no notebooks etc. without **ITO**!
  - Ionic conductors (liquid and solid)

    Batteries, fuel cells, sensors, ...
  - Conductors for high temperature applications; corrosive environments, ..
    (Graphite, Silicides, Nitrides, ...)

    Example: **$MoSi_2$** for heating elements in corrosive environments (dishwasher!).
  - Organic conductors (and semiconductors)

    The future High-Tech key materials?

- Numbers to know (order of magnitude accuracy sufficient)

ρ(decent metals) about **2 μΩcm**
ρ(technical semiconductors) around **1 Ωcm**
ρ(insulators) > 1 GΩcm

- No electrical engineering without conductors!
  - Hundreds of specialized metal alloys exist just for "wires" because besides σ, other demands must be met, too:

    Money, Chemistry (try **Na**!), Mechanical and Thermal properties, Compatibility with other materials, Compatibility with production technologies, ...

  - Example for unexpected conductors being "best" compromise:

    Poly Si, Silicides, **TiN**, **W** in integrated circuits

- Don't forget Special Applications:

  Contacts (switches, plugs, ...);
  Resistors;
  Heating elements; ...

- Thermionic emission provides electron beams.
  The electron beam current (density) is given by the *Richardson equation*:

$$j = A \cdot T^2 \cdot \exp - \frac{E_A}{kT}$$

  - A_{theo} **= 120 A · cm^{-2} · K^{-2}** for free electron gas model
    A_{exp} ≈ **(20 - 160) A · cm^{-2} · K^{-2}**
  - E_A **=** work function ≈ **(2 - >6) eV**
  - Materials of choice: **W**, **LaB$_6$** single crystal
- High field effects (tunneling, barrier lowering) allow large currents at low ***T*** from small (nm) size emitter

  Needs **UHV**!

- There are several thermoelectric effects for metal junctions; always encountered in non-equilibrium.
  - *Seebeck effect:*
    Thermovoltage develops if a metal A-metal B junction is at a temperature different form the "rest", i.e. if there is a temperature gradeient

    Essential for measuring (high) temperatures with a "thermoelement"
    Future use for efficient conversion of heat to electricity ???

  - *Peltier effect:*
    Electrical current *I* through a metal - metal (or metal - semiconductor) junction induces a temperature gradient ∝ *I*, i.e. one of the junction may "cool down".

    Used for electrical cooling of (relatively small) devices. Only big effect if electrical heating (∝ I^2) is small.

- Electrical current can conducted by *ions* in
  - Liquid electrolytes (like **H_2SO_4** in your "lead - acid" car battery); including gels
  - Solid electrolytes (= ion-conducting crystals). Mandatory for fuel cells and sensors
  - Ion beams. Used in (expensive) machinery for "nanoprocessing".

  Challenge: Find / design a material with a "good" ion conductivity at room temperature

- Basic principle
  - *Diffusion current* j_{diff} driven by concentration gradients **grad(*c*)** of the charged particles (= ions here) equilibrates with the
  - *Field current* j_{field} caused by the internal field always associated to concentration gradients of charged particles plus the field coming from the outside

$$j_{diff} = -D \cdot \text{grad}(c)$$

$$j_{field} = \sigma \cdot E = q \cdot c \cdot \mu \cdot E$$

- Diffusion coefficient ***D*** and mobility **μ** are linked via theEinstein relation;
  concentration ***c*(*x*)** and potential ***U*(*x*)** or field ***E*(*x*) = –d*U*/d*x***by the Poisson equation.

$$\mu = eD/kT$$

$$-\frac{d^2U}{dx^2} = \frac{dE}{dx} = \frac{e\cdot c(x)}{\epsilon\epsilon_0}$$

Immediate results of the equations from above are:

- In equilibrium we find a preserved quantity, i.e. a quantity independent of ***x*** - the electrochemical potential V_{ec}:

$$V_{ec} = \text{const.} = e\cdot U(x) + kT\cdot\ln c(x)$$

- If you rewrite the equaiton for ***c*(*x*)**, it simply asserts that the particles are distributed on the energy scale according to the Boltzmann distrubution:

$$c(x) = \exp -\frac{(Vx) - V_{ec}}{kT}$$

- Electrical field *gradients* and concentration *gradients* at "contacts" are coupled and non-zero on a length scale given by the **Debye length d_{Debye}**
- The Debye length is an extremely important material parameter in "**ionics**" (akin to the space charge region width in semiconductors); it depends on temperature ***T*** and in particular on the (bulk) concentration c_0 of the (ionic) carriers.
- The Debye length is not an important material parameter in metals since it is so small that it doesn't matter much.

$$d_{Debye} = \left(\frac{\epsilon\cdot\epsilon_0\cdot kT}{e^2\cdot c_0}\right)^{1/2}$$

The potential difference between two materials (her ionic conductors) in close contact thus...

- ... extends over a length given (approximately) by :

$$d_{Debye}(1) + d_{Debye}(2)$$

- ... is directly given by the Boltzmann distribution written for the energy:
  (with the c_i =equilibrium conc. far away from the contact.
- The famous *Nernst equation*, fundamental to ionics, is thus just the Boltzmann distribution in disguise!

$$\frac{c_1}{c_2} = \exp -\frac{e\cdot\Delta U}{kT}$$

Boltz-mann

$$\Delta U = -\frac{kT}{e}\cdot\ln\frac{c_1}{c_2}$$

Nernst's equation

"Ionic" sensors (most famous the ZrO_2 - based O_2 sensor in your car exhaust system) produce a voltage according to the Nernst equation because the concentration of ions on the exposed side depends somehow on the concentration of the species to be measured.

Questionaire

All multiple choice questions zu 2. Conductors