5.6 Calculation of the canonical ensemble

Here we could repeat the calculation of the last section just with one less restriction, since for all micro states $N_i = N$ holds. This simplification we can take directly into account in the grand canonical partition function of Eq. (5.46)

$$Z_{GC} = \sum_i \exp(-\beta U_i - \gamma N_i) = \sum_i \exp(-\beta U_i - \gamma N) = \exp(-\gamma N) \sum_i \exp(-\beta U_i) := \exp(-\gamma N) Z_C \quad , \tag{5.56}$$

with the definition of the canonical partition function

$$Z_C = \sum_i \exp(-\beta U_i) \quad . \tag{5.57}$$

This result we can directly incorporate into Eq. (5.55)

$$\Omega(T, V, \mu) = -kT \ln(Z_C(T, V, \mu)) - kT \ln(\exp(-\gamma N)) = -kT \ln(Z_C(T, V, \mu)) - \mu N \quad . \tag{5.58}$$

Comparison with Eq. (5.54) directly gives

$$-kT \ln(Z_C) = U - TS = F \quad . \tag{5.59}$$

Of course the variable μ has to be replaced by its dependence of N leading to the standard definition of the free energy $F(T, V, N)$ and showing that Ω and F are Legendre transformed with respect to the pair $(N \leftrightarrow \mu)$.