Christian-Albrechts-Universität

INVESTIGATION OF ERROR SUSCEPTIBILITY OF NOISY SOLITON PULSES IN THE NONLINEAR FREQUENCY DOMAIN

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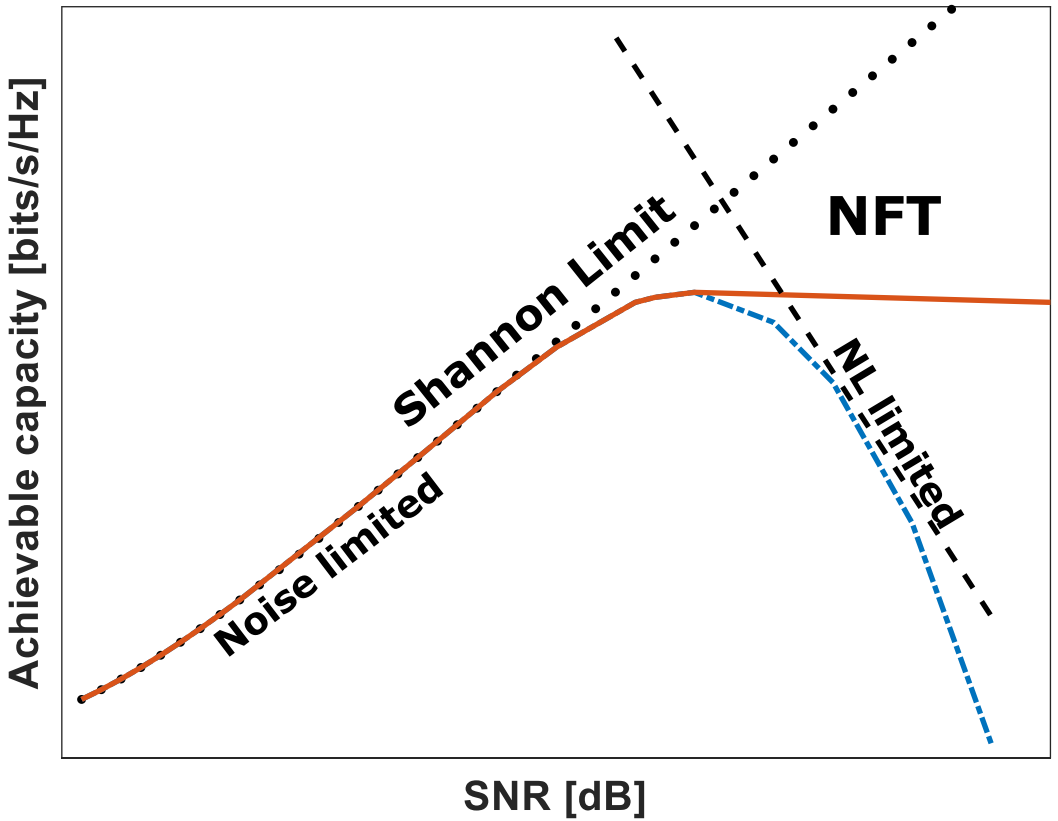
17.02.2017

Outline

- Motivation
- Simulation Model
- Numerical Methods for the Nonlinear Fourier Transform
- Some Insights from the Simulation
- Summary

Motivation

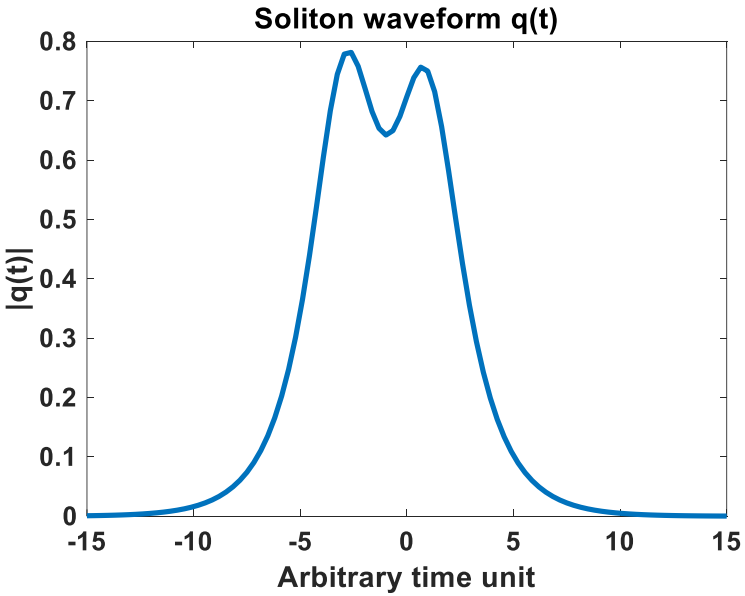


See Essiambre et al.
JLT, Vol. 28, 2010

The Nonlinear Fourier Transform (NFT) has the potential to increase the achievable rate at higher SNR.

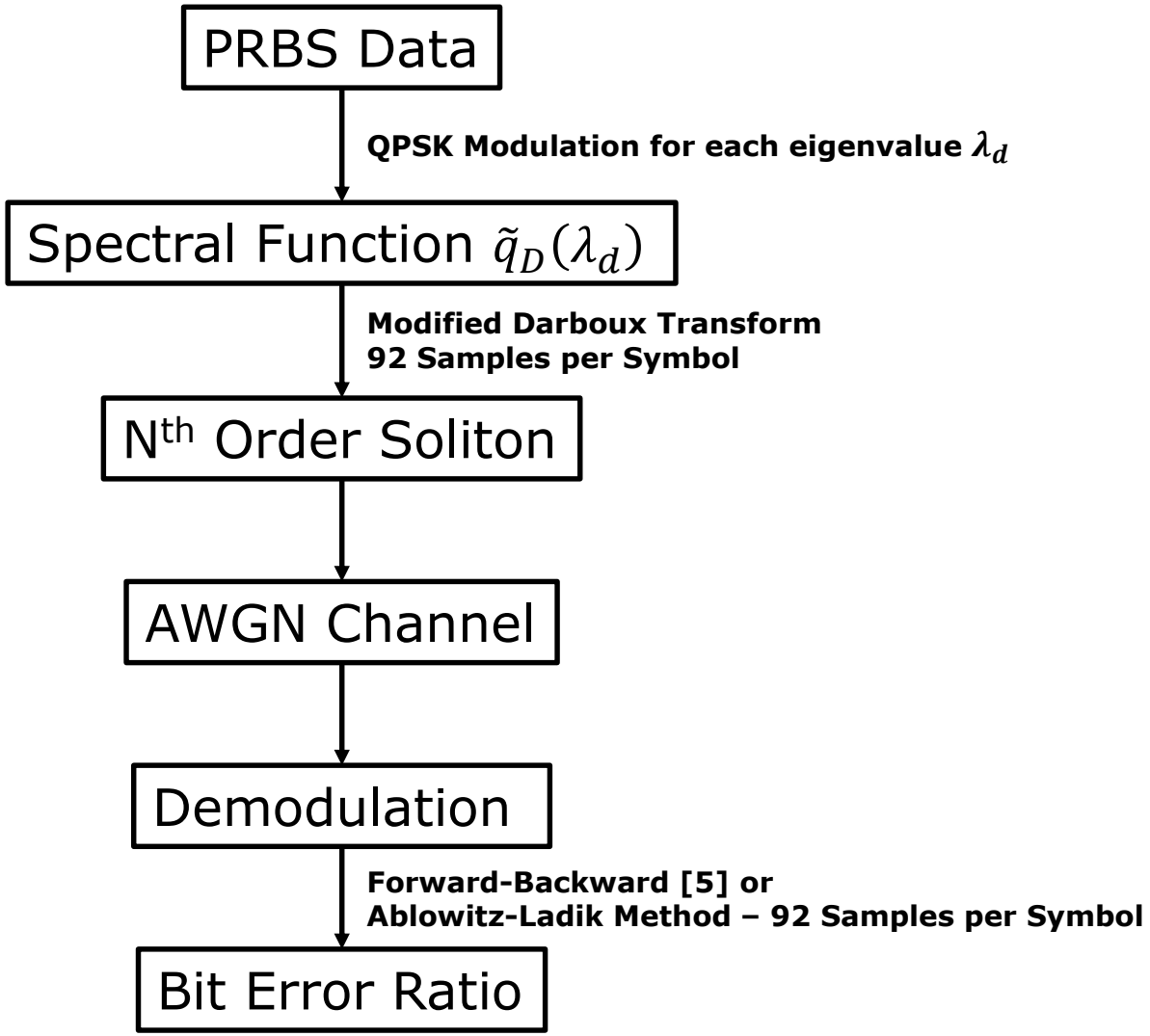
Motivation – Nonlinear Fourier Transform

- Nonlinear Fourier Transform (NFT)
  - For integrable systems the time signal can be described with two spectra.
    - Invariant continuous spectrum $\hat{q}_C(\lambda)$, $\lambda \in \mathbb{R}$
    - Invariant discrete spectrum $\tilde{q}_D(\lambda_d)$, $\lambda_d \in \mathbb{C}^+, d = 1,2,\ldots$
    - Generalized frequency λ (eigenvalue)
  - The evolution inside the optical fiber is just a linear phase shift.

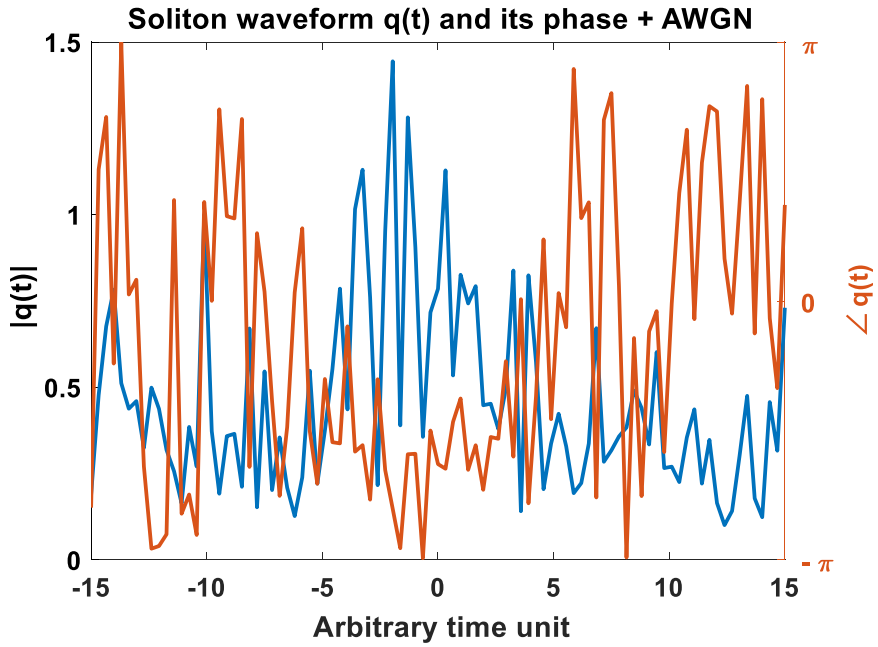


- Observation:
  - **Some modulated soliton pulse shapes are more error prone.** [3]

Simulation Model



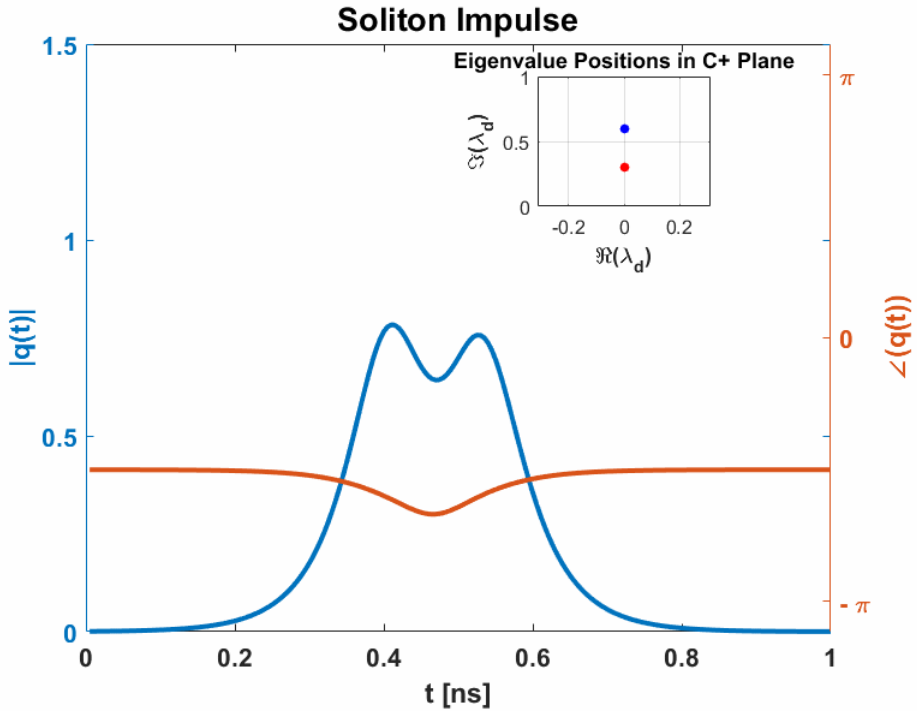
- Changeable Parameters
  - $\tilde{q}_D(\lambda_d) = A \exp\left(j\left(k_d \frac{\pi}{2} + p_d\right)\right)$
  - Gap between $\lambda_{d=1\ldots N}$
  - Additional phase difference for each QPSK modulation of $\tilde{q}_D(\lambda_d)$



Numerical Methods at the Transmitter Side

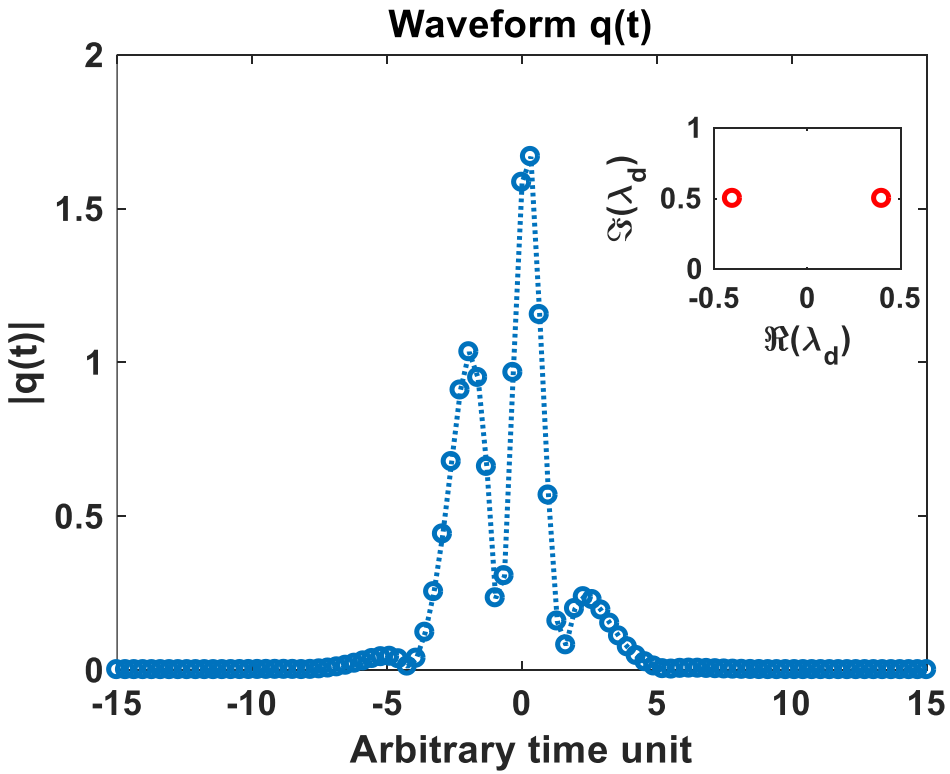
- Inverse Nonlinear Fourier Transform
  - Continuous spectrum $\hat{q}_C(\lambda) = 0$

- Darboux Transform (DT) for generating solitons
  - Starting from a known solution of the Nonlinear Schrödinger Equation (NLSE) (e. g.: $q(t) = 0$)
  - DT can generate iteratively with a given eigenvector $\vec{v}_0 = (A\ exp(-j\lambda_d t), B\ exp(j\lambda_d t))^{T}$[1] new eigenvectors for each λ_d to create a soliton impulse.
  - A and B are normalization factors.
    e.g. [5]: $A = 1, B = -\frac{\tilde{q}(\lambda_d)}{\lambda_d - \lambda_d^*} \prod_{k=1, k\neq d}^{M} \frac{\lambda_d - \lambda_k}{\lambda_d - \lambda_k^*}$



Numerical Methods at the Receiver Side

- Objective is to find the nonlinear Fourier coefficients for the two spectra.
  - We are looking at the scattering behaviour of an arbitrary wave on the signal from $-\infty < t < \infty$.
- Ablowitz-Ladik Method (AL)
  - Integrable discretization of NLSE
  - Z – Transform of λ to preserve the quantities (energy, momentum)



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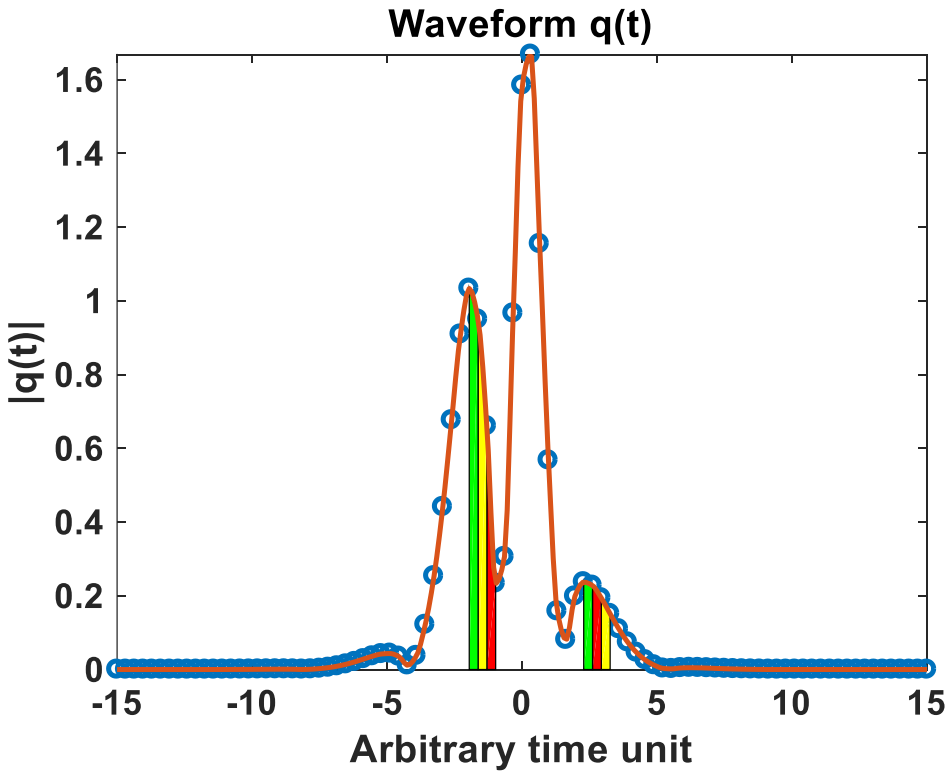
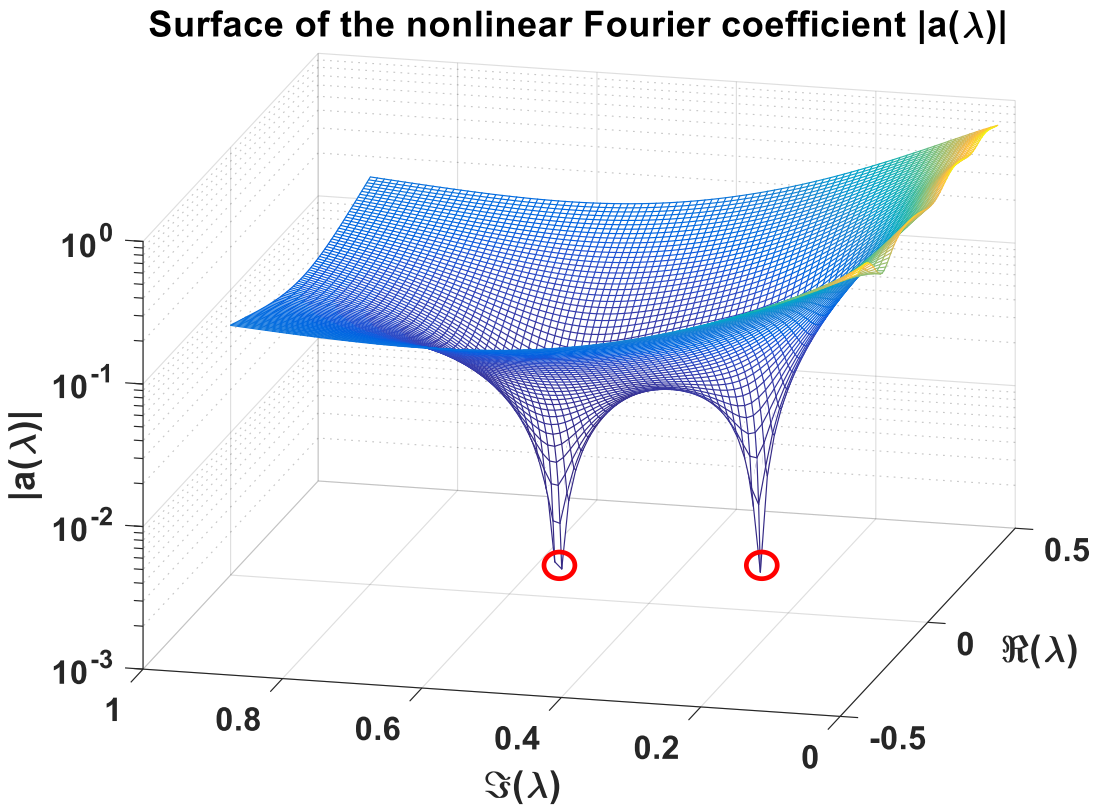
- Ablowitz-Ladik Method (AL)
  - Integrable discretization of NLSE
  - Z – Transform of λ to preserve the quantities (energy, momentum)
- Forward-Backward Method (FB)[5]
  - Splitting the scattering matrix S into two matrices R $(-\infty \to 0)$ and L $(+\infty \to 0)$
  - Similar to AL a numerical approximation of the pulse is needed. (e.g. Trapezoid Rule)



Methodology of the Demodulation

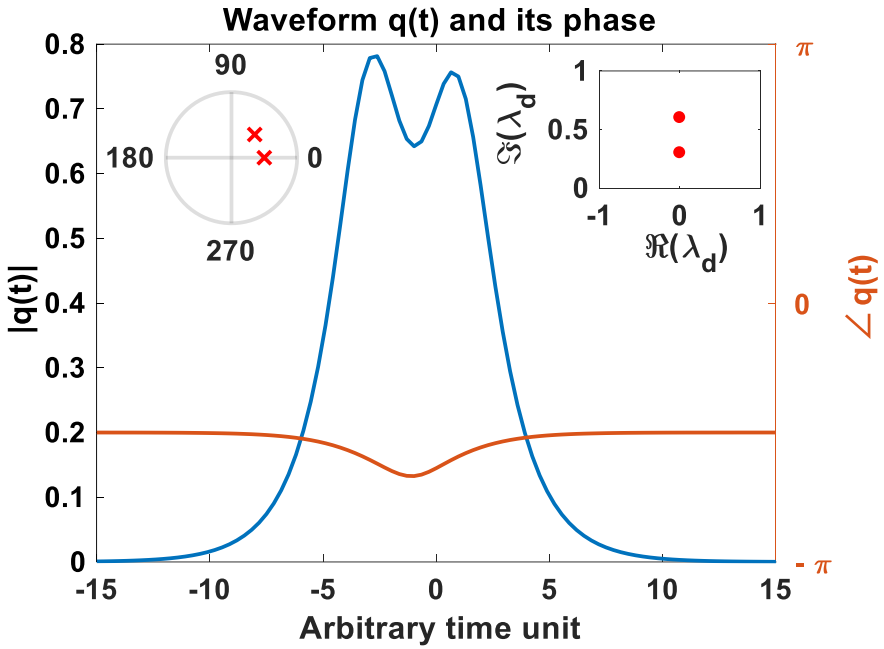
- Assumption: Continuous spectrum $\hat{q}_C(\lambda) = 0$

1. Transform $q(t,z)$ into the nonlinear spectrum
2. Find λ_d
3. Calculate the spectral function $\tilde{q}_D(\lambda_d)$
4. Data Recovery

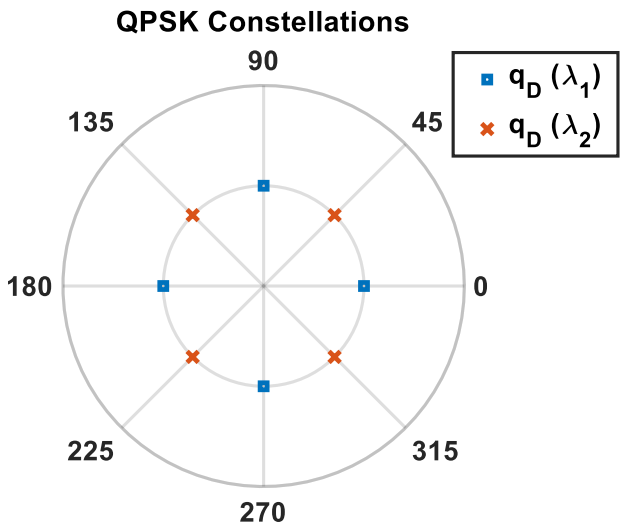


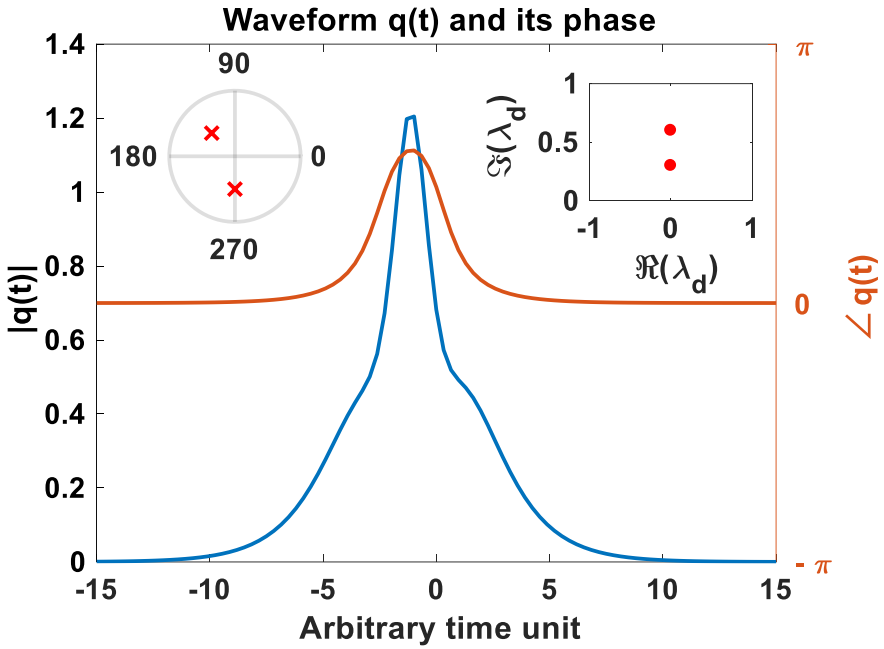
Transmitter – Generating Modulated Soliton Pulses

- E.g. (10.000 Symbols or 100 Errors) [3]:
  - $\tilde{q}_D(\boldsymbol{\lambda_1 = 0.3j}) = exp\left(jk_1\frac{\pi}{2}\right)$; $\tilde{q}_D(\boldsymbol{\lambda_2 = 0.6j}) = exp\left(j\left(k_2\frac{\pi}{2} + \boldsymbol{\frac{\pi}{4}}\right)\right)$ $with$ $k_{1,2} = 0,1,2,3$



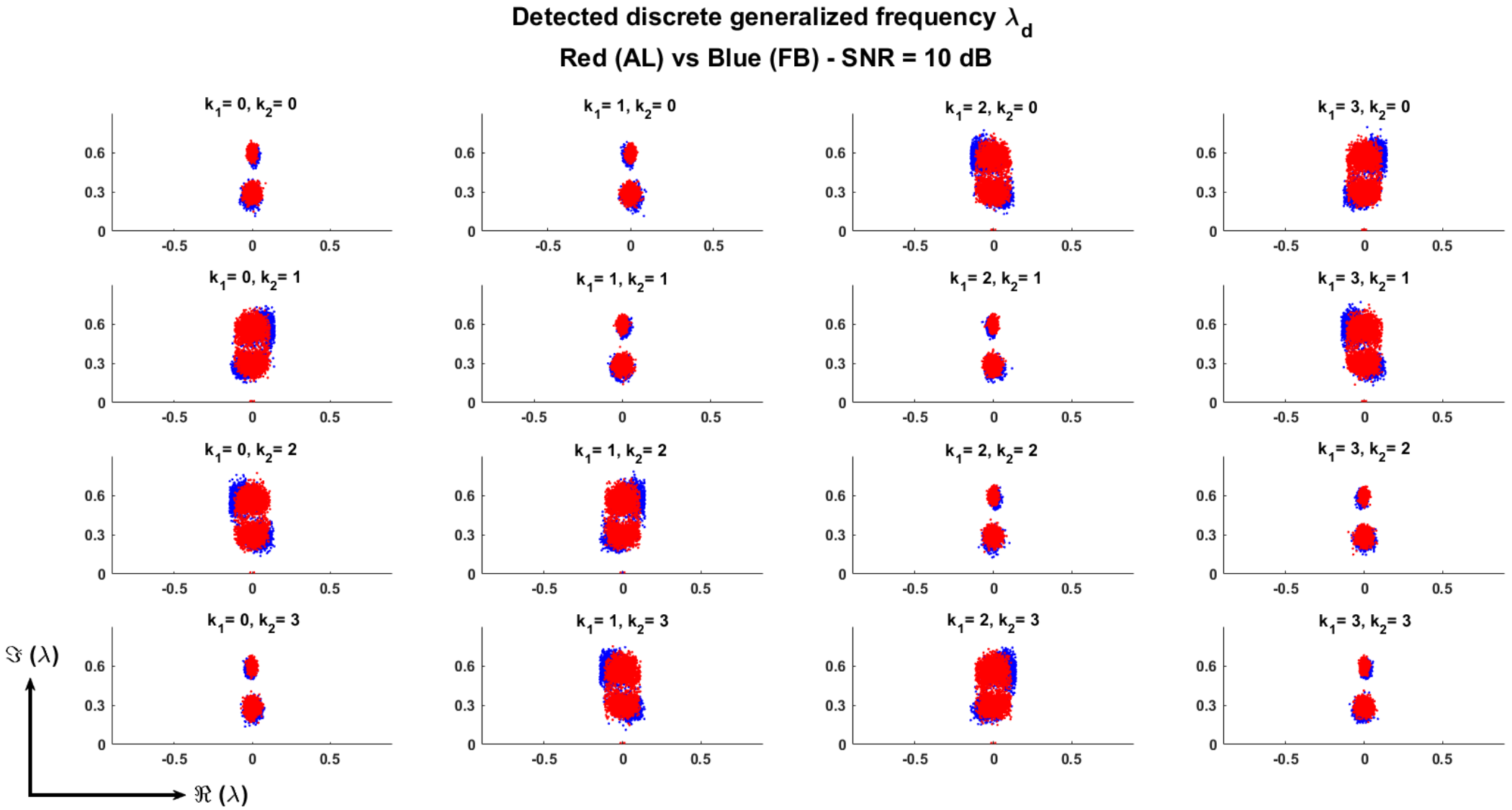
Advantageous pulse shape
for eigenvalue detection





Less advantageous pulse shape
for eigenvalue detection

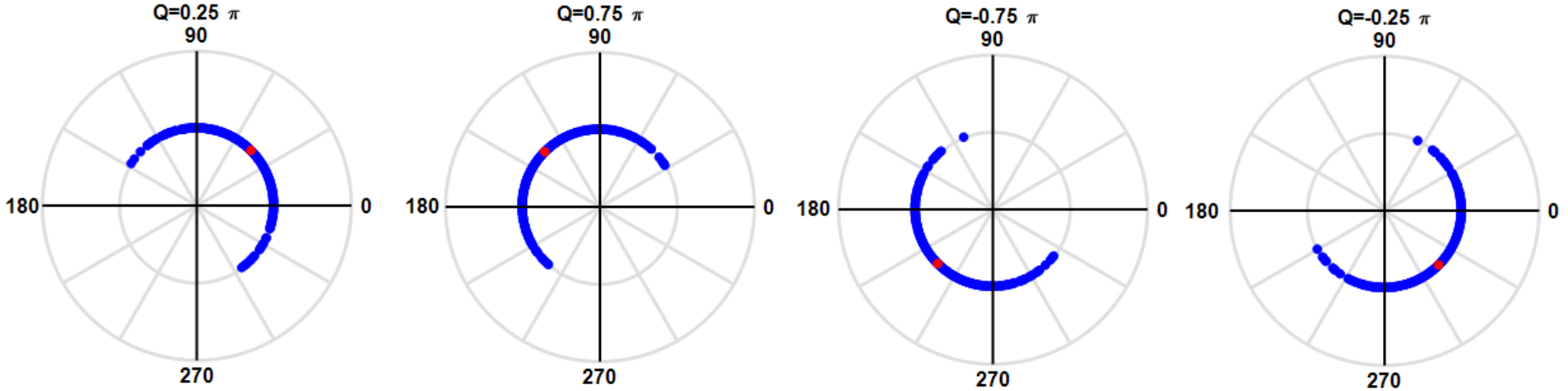
Detection of Eigenvalues



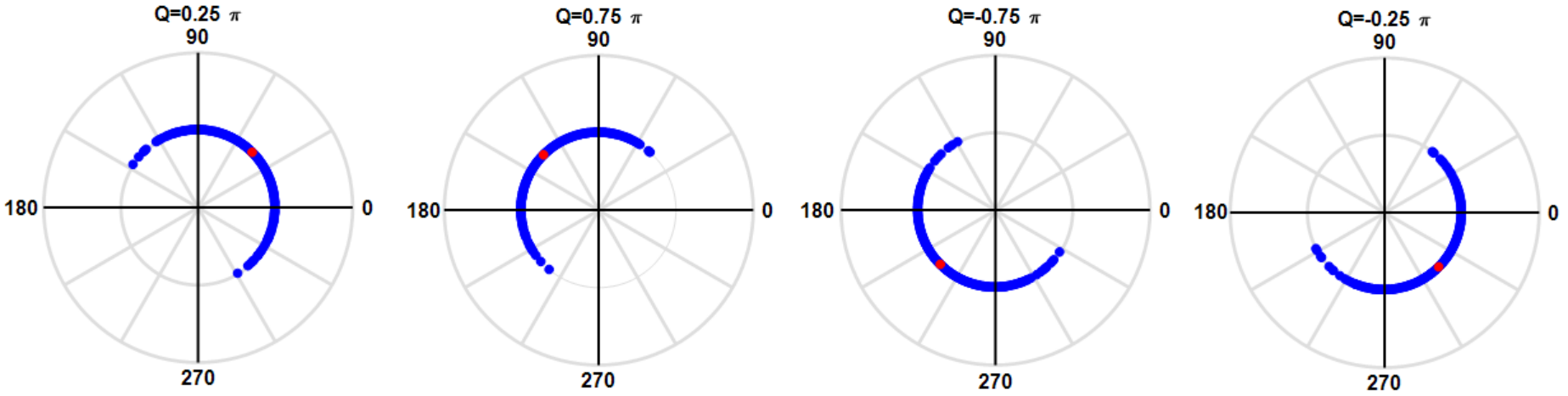
Detected Constellation of 1st Eigenvalue

- SNR = 10 dB
- $\lambda_d = 0.6j$

Ablowitz-Ladik



Foward-Backward

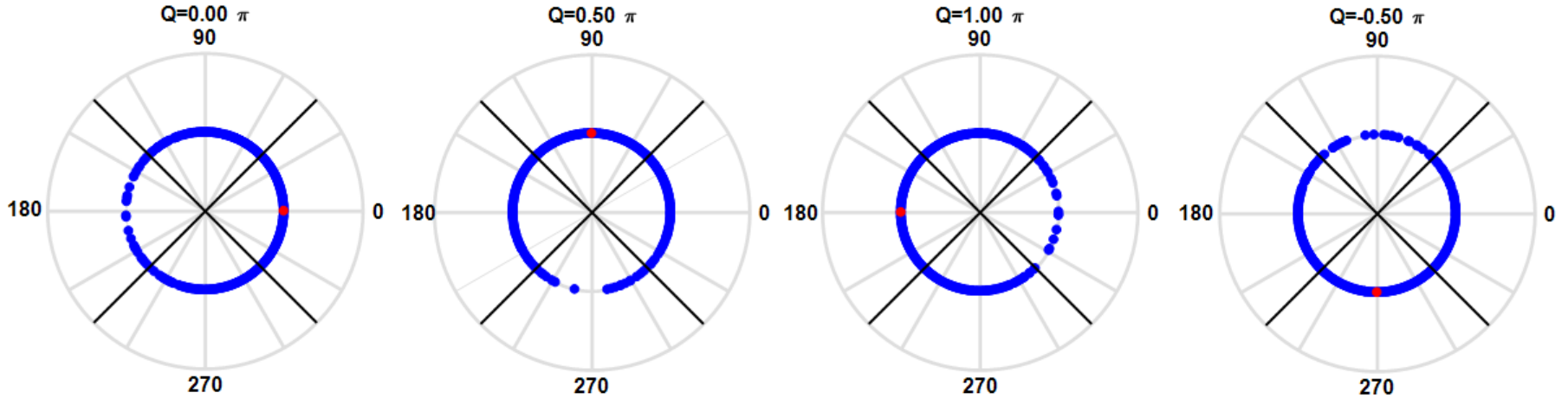


Ablowitz-Ladik and Forward-Backward show similar behaviour.

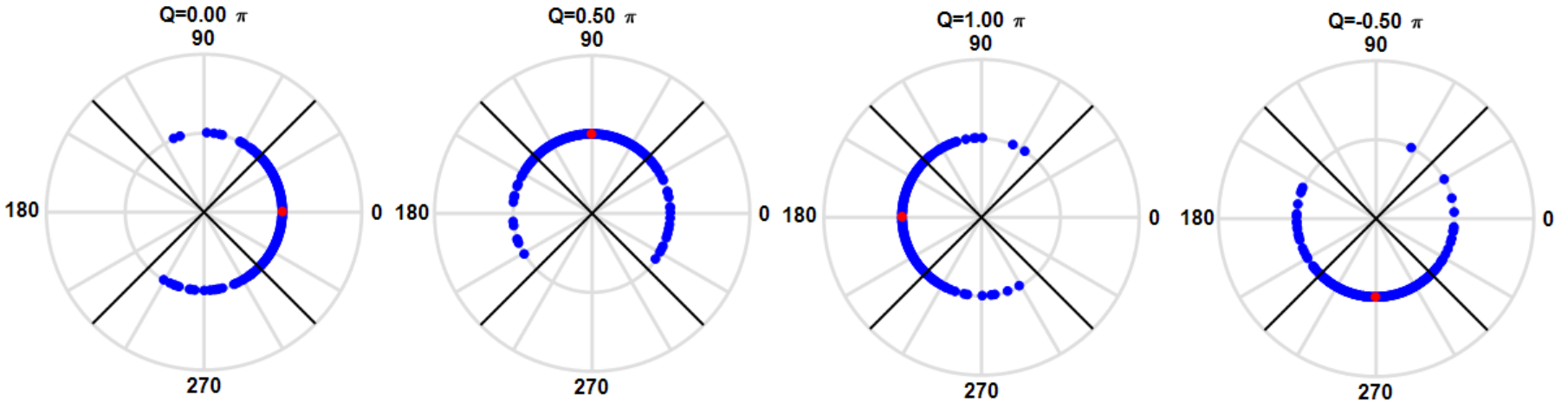
Detected Constellation of 2nd Eigenvalue

- SNR = 10 dB
- $\lambda_d = 0.3j$

Ablowitz-Ladik

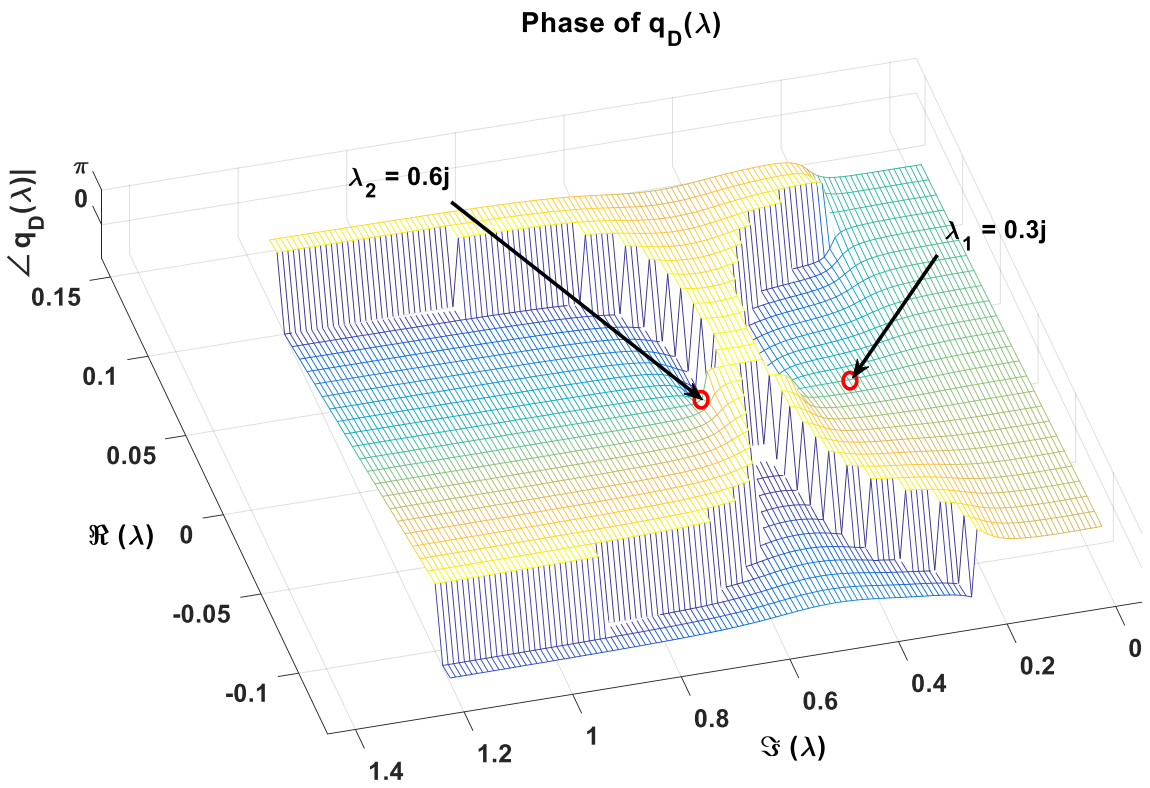


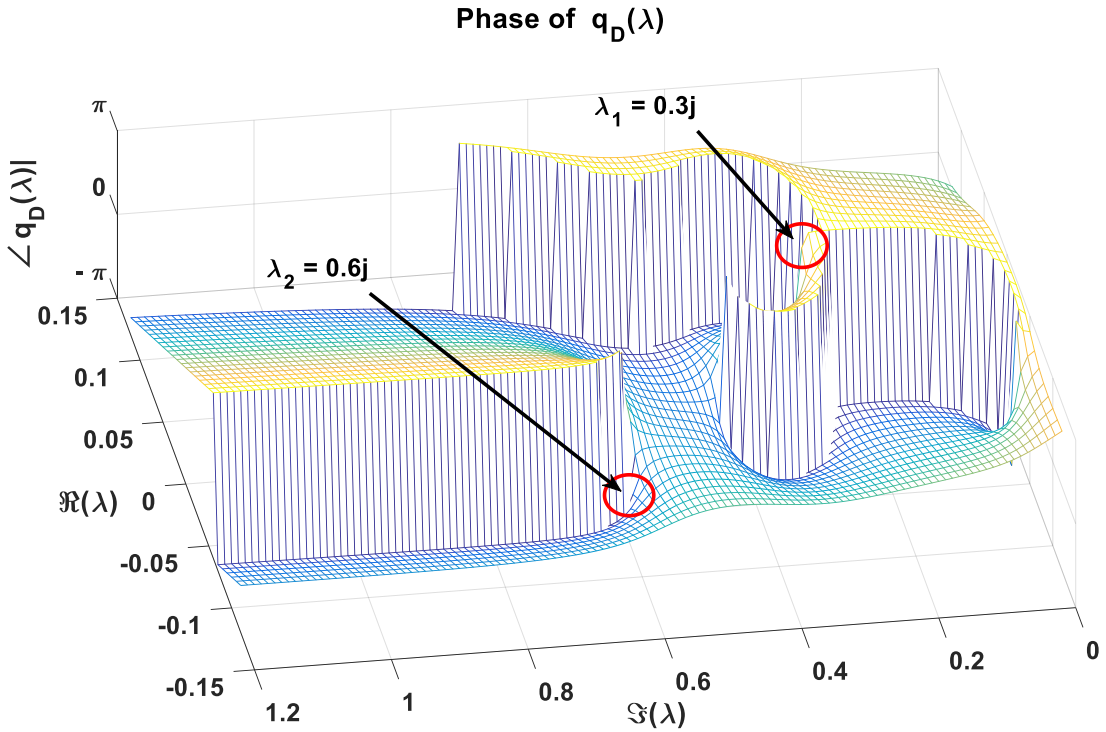
Foward-Backward

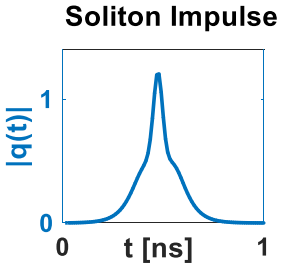


Ablowitz-Ladik shows worse behaviour than Foward-Backward.

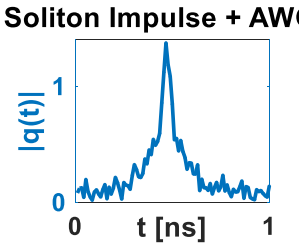
Analysis of the Phase (Ablowitz-Ladik Method)







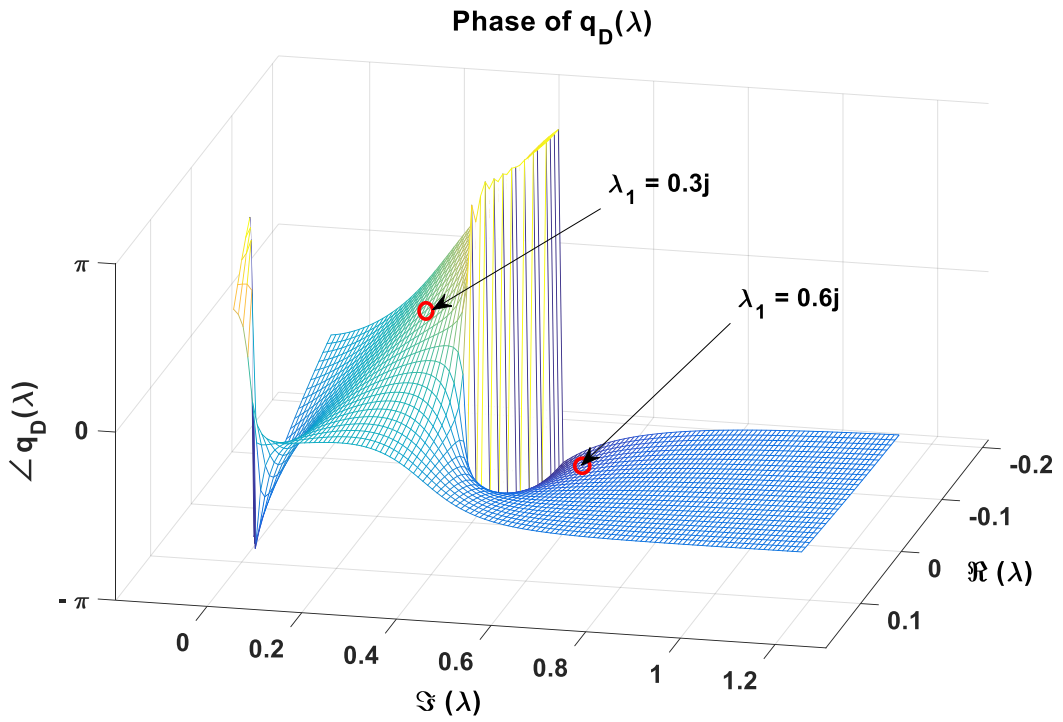
Noiseless

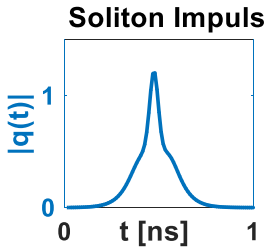


With AWGN SNR = 10 dB

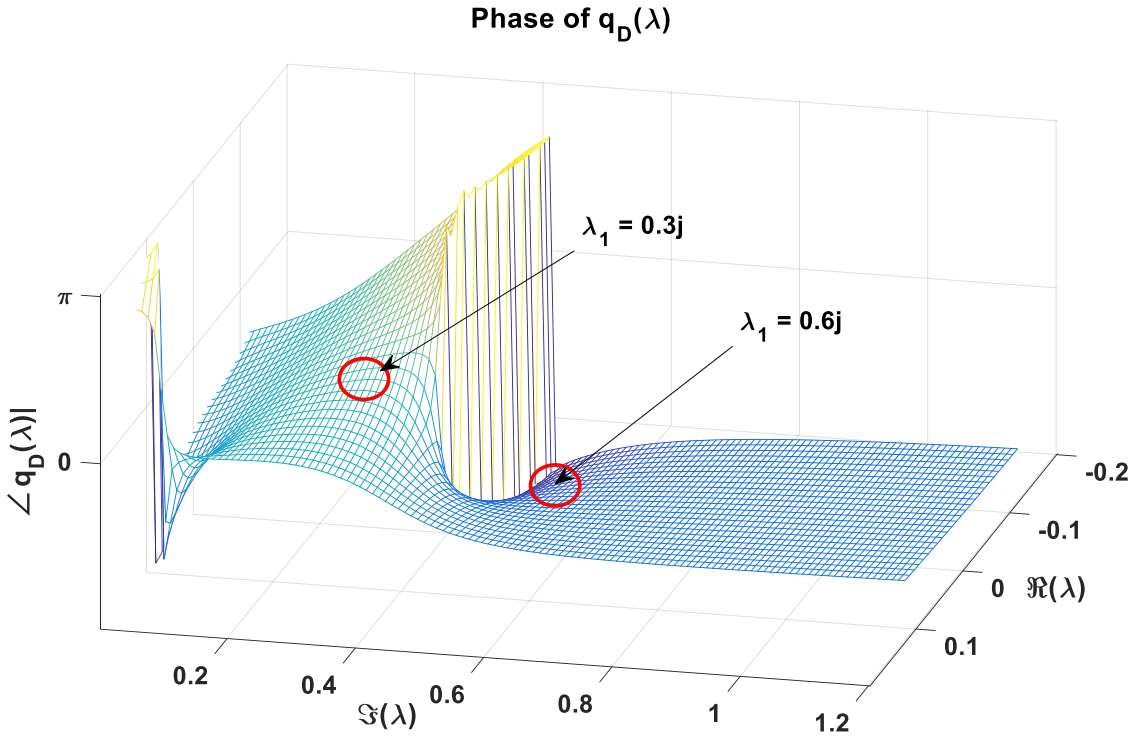
The eigenvalues change their positions because of the noise.

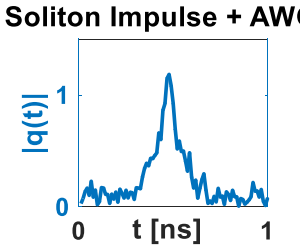
Analysis of the Phase (Forward-Backward Method)





Noiseless

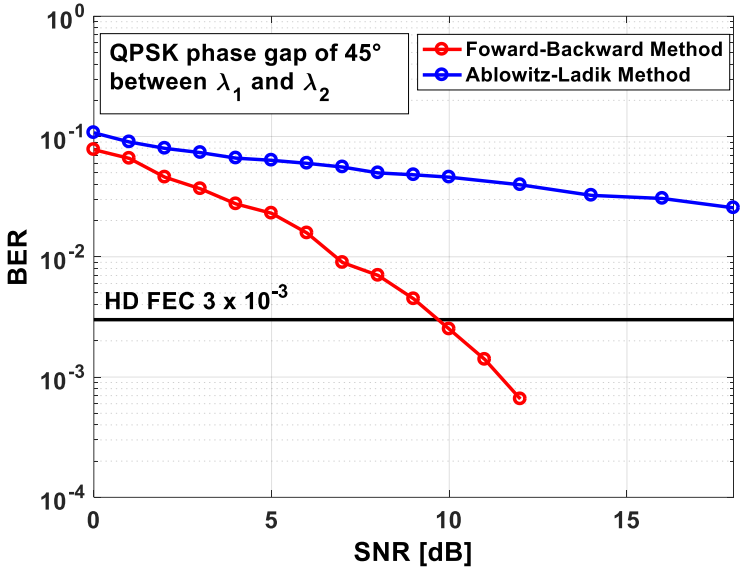




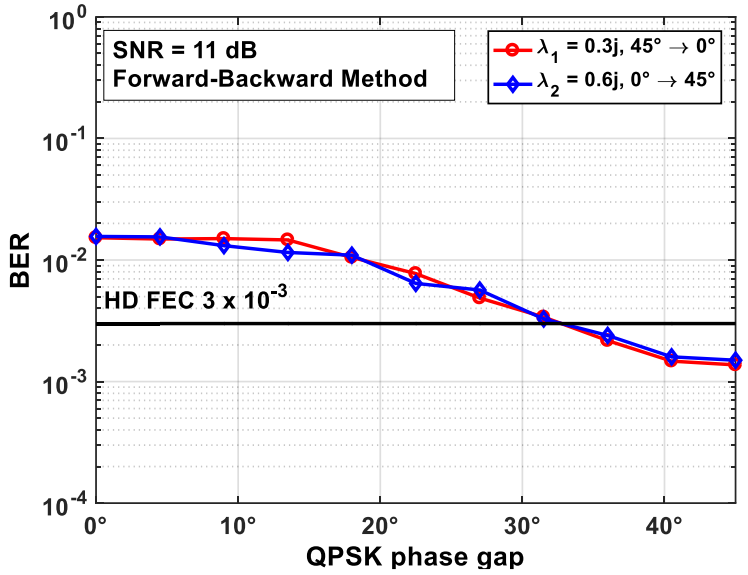
With AWGN SNR = 10 dB

The spectral function is more stable with the Forward-Backward Method.

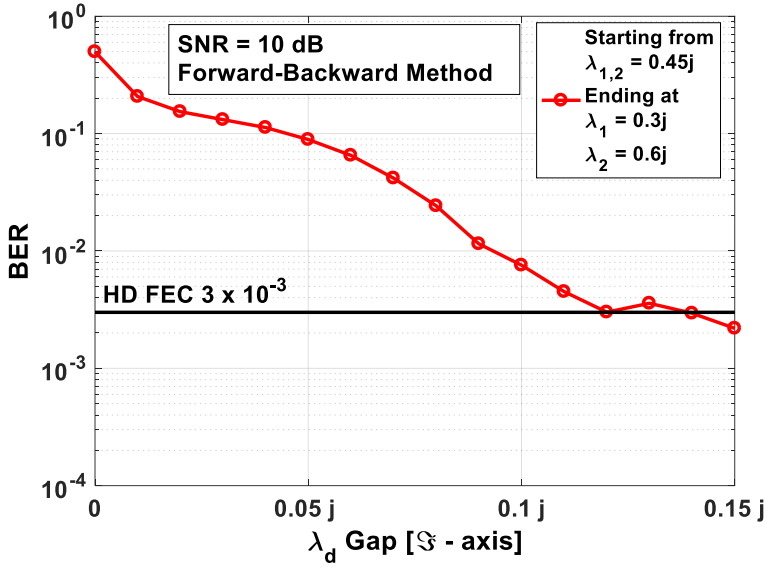
BER Analysis



Forward-Backward Method yields a better BER performance than Ablowitz-Ladik Method.



A minimum of 30° QPSK phase difference for each eigenvalue is needed for sufficient BER performance.



A gap of at least 0.3j for each eigenvalue is needed to stay below the HD FEC threshold.

Summary

- Ablowitz-Ladik Discretization yields better detection of the eigenvalues.
- Forward-Backward Method gives more stable spectral functions.
- A phase difference between the QPSK modulation of each eigenvalue yields better BER performance.
- The temporal width of the soliton seems to be the main issue for the error susceptibility.

Thank you!

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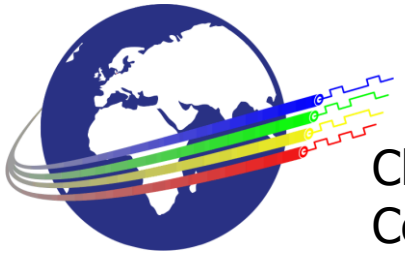

Chair of
Communications

Some References

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2. M.I Yousefi, „Information Transmission Using the Nonlinear Fourier Transform“, Ph.D. disseration, University Toronto, Nov. 2012

3. V. Aref, H. Bülow, et al., „Experimental Demonstration of Nonlinear Frequency Division Multiplexed Transmission“, European Conf. Opt. Commun., September 2015

4. V. Aref, H. Bülow, et al., „Transmission of waveforms determined by 7 eigenvalues with PSK-Modulated Spectral Amplitudes“, European Conf. Opt. Commun., September 2016

5. V. Aref, „Control and Detection of Discrete Spectral Amplitudes in Nonlinear Fourier Spectrum“, Arxiv.org Preprint, 2016