Flatness Based Speed Control of Drive Systems with Resonant Loads

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Abstract—The design and analysis of a flatness based speed control method for drive systems with elastically coupled loads is presented. Drive systems with elastically coupled loads tend to mechanical vibrations due to a finite stiffness of the drive shafts material. These unwanted vibrations stress the system and can reduce its lifetime. The way out and aim of this analysis is a high performance speed control of drive systems with resonant loads with damping of mechanical oscillations. Conventional control methods like proportional-integral based controllers are not able to damp these oscillations effectively. Model based control methods are more appropriate to handle this problem. Therefore, the rather new model based control concept based on differential flatness is applied to the considered system. High dynamic and suppression of mechanical oscillations can be achieved with flatness based control without the reduction of the disturbance reaction. Furthermore, only measurement of the motor speed is required. Simulation and measurement results confirm these statements. The results are compared to conventional PI-control.

1. INTRODUCTION

Conventional variable speed drive systems consist of an inverter-fed ac motor and a load. The load is coupled via mechanical transmission elements, e.g. shafts, gears and couplings, which have a non ideal transfer behavior like a finite torsional stiffness. The finite stiffness of the mechanical parts reduces the drive system performance, can lead to unwanted torsional oscillations and stresses both mechanical and electrical components of the drive system. These problems occur for example in rolling mill drives [1], [2], windmill applications [3], electric vehicles [4], [5], servo drives [6] and conveyor drives [7].

To avoid mechanical oscillations, the behavior of the elastic elements has to be considered in the design process of the speed control.

The two-inertia model, as can be seen in Fig. 1, is the simplest model of elastically coupled drive systems [2]. In many cases it is sufficient for the control design to reduce a multi-mass drive system with one dominant resonant frequency to a two-mass model [8]. Normally, conventional PI controller are used in industrial applications for speed control of resonant drive systems [1], [9]. Various design methods for tuning the PI control parameters with the aim to reduce torsional vibrations have been presented, e.g. in [9] and [10]. However, PI control method without additional feedback provides constricted pole-placement freedom to damp mechanical vibrations effectively [11].

One possibility to reduce torsional oscillations is the feedback of additional system states. A systematic analysis of speed control with different additional feedbacks is presented in [12]. The feedback of all system states, referred to as state space control, yields to a theoretically free pole placement of the closed speed control loop and thus to a free choice of the system dynamics. State space speed control for drive system with resonant loads has been analyzed in [1], [13]–[17].

Other possible control methods which do not need necessarily additional feedbacks are model based or nonlinear control methods such as sliding mode control, \( H_\infty \) control, model based predictive control or flatness based control (FBC). Sliding mode control for elastically coupled drive systems has been analyzed in [7], [18]–[20]. \( H_\infty \) control for position control of two-mass systems is presented in [21]. Model based predictive control for speed control of drive systems with resonant loads has been presented in [11], [22]–[24]. A detailed comparative study of conventional PI control, state space control and model based predictive control is presented in [25].

Flatness based control for induction machines has been analyzed in [26]–[30]. Elastically coupled loads and the influence on the control performance are not considered in these publications.

The aim of this paper is the design and the analysis of a flatness based speed control considering elastically coupled loads. As far as the authors know, this paper presents a flatness based speed control concept for elastically coupled two-mass systems for the first time. The results will be compared to the conventional PI-based speed control.

The analysis is structured as follows. A system description is given in section II. In section III a transformation from a common state space model into controller form is shown which is used for the derivation of the flatness based control in section IV. Simulation and measurement results are presented in section V and VI, respectively. The paper is finished by a conclusion.
II. SYSTEM DESCRIPTION

A typical variable speed drive system containing gears, elastic couplings and loads with additional inertia is represented in Fig. 2. The driving motor is fed by a PWM inverter to create variable currents. Field oriented control is used for current and flux control. The motor is connected to a load via gearbox (transmission ratio 1), a long torsional driveshaft and an additional inertia. The considered system can be divided into an electrical and a mechanical part. Whereas the time constant of the electrical part is much smaller than the time constant of the mechanical part, thus, the electrical part is neglected for the speed control design.

![Fig. 2. Topology of the drive system](image)

The mechanical part is modeled as a two-inertia system as shown in Fig. 3. The shaft is represented with the torsional elasticity $c_T$ and the internal damping $d_T$. The two-mass model parameters are given in Table I. Consequently, the ratio $R$ of load side inertia to drive side inertia and the resonance frequency $f_{res}$ are as follows:

$$R = \frac{J_L}{J_M} = 3.87$$

$$f_{res} = \frac{1}{2\pi} \sqrt{\left(\frac{J_M + J_L}{J_M J_L}\right) \cdot c_T} \quad [1]$$

The transfer function of the mechanical part is given by:

$$G(s) = \frac{N_M}{M_M} = \frac{s - \frac{d_T}{J_M J_L} + s^2 \frac{1}{J_M J_L}}{s \left(\frac{J_M + J_L}{J_M J_L}\right) + \frac{1}{J_M J_L} + s^3}$$

The state space model is given by:

$$\dot{\xi} = A_\xi \xi + b_\xi u$$

$$y = c_\xi^T \xi$$

$$\begin{bmatrix} \dot{N}_M \\ \dot{M}_S \\ \dot{N}_L \end{bmatrix} = \begin{bmatrix} 0 & -c_T & 0 \\ c_T & \frac{1}{J_L} & -c_T \\ 0 & \frac{1}{J_L} & 0 \end{bmatrix} \begin{bmatrix} N_M \\ M_S \\ N_L \end{bmatrix} + \frac{1}{J_M J_L} \begin{bmatrix} \frac{d_T}{J_M} \\ 0 \end{bmatrix}$$

$$J_M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Where $N_M$ is the motor speed and $N_L$ the load speed. $M_S$ and $M_M$ are the torsional shaft torque and electromagnetic torque of the motor, respectively.

![Fig. 3. Block diagram of two-inertia system](image)

III. CONTROLLER FORM

For the derivation of the flatness based control, the mechanical system is transformed from the common state space model (3) into controller form (5) [31]:

$$\dot{\xi} = A_\xi \xi + b_\xi u$$

$$y = c_\xi^T \xi$$

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -a_0 & 0 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} \cdot \xi$$

Comparing the coefficients of the transfer function (2) with the coefficients of the transfer function (6):

$$G(s) = \frac{b_0 + s \cdot b_1 + s^2 \cdot b_2}{a_0 + s \cdot a_1 + s^2 \cdot a_2 + s^3}$$

yields to the following parameter of the system model in controller form:

$$a_0 = 0$$

$$b_0 = \frac{c_T}{J_M J_L}$$

$$a_1 = \frac{(J_M + J_L) c_T}{J_M J_L}$$

$$b_1 = \frac{d_T}{J_M J_L}$$

$$a_2 = \frac{(J_M + J_L) d_T}{J_M J_L}$$

$$b_2 = \frac{1}{J_M J_L}$$

The transformation matrix $T$ for the transformation into the controller form $\xi = T^{-1} \tilde{\xi}$ is given by:

$$T = \begin{bmatrix} \frac{c_T}{J_M J_L} & \frac{d_T}{J_M J_L} & \frac{1}{J_M J_L} \\ \frac{d_T}{J_M J_L} & \frac{c_T}{J_M J_L} & \frac{d_T}{J_M J_L} \\ \frac{d_T}{J_M J_L} & \frac{d_T}{J_M J_L} & \frac{c_T}{J_M J_L} \end{bmatrix}$$

Neglecting the internal damping $d_T$ the state vector in controller form can be derived by:

$$\xi = \begin{bmatrix} J_M J_L N_L \\ J_M (N_M - N_L) \end{bmatrix}$$

The system description in controller form yields to a simple derivation of the FBC as will be shown in the next section.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Notation</th>
<th>Value</th>
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<tr>
<td>Drive side data:</td>
<td>Inertia</td>
<td>$J_M$</td>
</tr>
<tr>
<td>Load side data:</td>
<td>Inertia</td>
<td>$J_L$</td>
</tr>
<tr>
<td>Drive shaft data:</td>
<td>Torsional stiffness</td>
<td>$c_T$</td>
</tr>
<tr>
<td></td>
<td>Internal damping</td>
<td>$d_T$ (estimated)</td>
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TABLE I

PARAMETERS OF TWO-INERTIA MODEL
IV. FLATTENESS BASED CONTROL

The feedforward control based on differential flatness of the system is a rather new control concept. Differentially flat nonlinear systems have been introduced by Fliess et al. [32] in 1992. An introduction with examples is given in [33]. The concept of flatness based control (FBC) allows to design the control as a combination of a nominal feedforward control and a feedback stabilizing control [34], as can be seen in Fig. 4. The flatness based feedforward control provides a nominal input trajectory \( u^*(t) \) which forces the system to the desired output trajectory \( y^*(t) \) in the nominal case [35]. This leads to one of the main advantages of the FBC approach, the command and the disturbance response can be designed separately.

A system given by:
\[
\dot{x} = f(x, u)
\]
with system state vector \( x = [x_1, \ldots, x_n]^T \) of order \( n \) and system input \( u = [u_1, \ldots, u_m]^T \) of order \( m \) is referred to as flat if and only if there exists a so-called flat output \( \bar{z} = [\bar{z}_1, \ldots, \bar{z}_m]^T \) of order \( m \):
\[
\bar{z} = \Phi (z, u, \dot{z}, \ldots, \dot{z}^{(\beta)})
\]
where \( \Phi \) is a smooth function of its arguments and \( \beta \) is chosen as reference trajectories due to a relatively free signal for the feedforward control. Possible trajectories are for example polynomials [37], Gevrey functions [38] or smoothing filters [30]. All of this reference trajectories are smooth enough because of the derivatives of the control variable. The transition time \( T \) is set depending on the desired final value \( y_T \) and the start value \( y_0 \).

The eigenfrequencies \( \omega_{z1} \) and \( \omega_{z2} \) result by system parameters and the selected damping coefficient \( D \):
\[
\omega_{z1/2} = \sqrt{\frac{J_M}{c_T} - 4D^2 + 4} \pm \sqrt{\frac{J_M}{c_T} - 4D^2}
\]
Die damping coefficient \( D = 1/\sqrt{2} = 0.707 \) is used for PI feedback stabilizing controller.

B. Feedback Stabilizing Controller

Proportional integral (PI) controller is used as feedback stabilizing controller which is needed to compensate deviations caused by disturbances and model uncertainties. The motor speed is used as control variable. Load speed and shaft torque do not have to be measured. The PI-controller can be tuned with several design methods. An appropriate method for PI-control of two-mass systems with ratio \( R \geq 2 \) is tuning the poles of the closed control loop to an identical damping coefficient [9]. For the design with identical damping coefficient \( D = D_1 = D_2 \), the following control parameters arises for a PI controller \( G_{PI}(s) = \frac{H_{PI}(s)}{N_{PI}(s)} = k_p + k_i/s \) [9]:
\[
k_p = 2JM_D (\omega_{z1} + \omega_{z2})
\]
\[
k_i = \frac{J_M J_L}{c_T} (\omega_{z1}^2 - \omega_{z2}^2)
\]
The eigenfrequencies \( \omega_{z1} \) and \( \omega_{z2} \) result by system parameters and the selected damping coefficient \( D \):
TABLE II
SYSTEM PARAMETERS

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Notation</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Drive motor IM</td>
<td>$P_{Drive}$</td>
<td>5.5 kW</td>
</tr>
<tr>
<td>Rated power</td>
<td>$P_{Drive}$</td>
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</tr>
<tr>
<td>Rated torque</td>
<td>$M_{Drive}$</td>
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</tr>
<tr>
<td>Rated speed</td>
<td>$N_{Drive}$</td>
<td>1455 rpm</td>
</tr>
<tr>
<td>Inertia</td>
<td>$J_{Drive}$</td>
<td>0.018 kgm²</td>
</tr>
<tr>
<td>Load motor</td>
<td>Servo IM</td>
<td></td>
</tr>
<tr>
<td>Rated power</td>
<td>$P_{Load}$</td>
<td>6.4 kW</td>
</tr>
<tr>
<td>Rated torque</td>
<td>$M_{Load}$</td>
<td>24.6 Nm</td>
</tr>
<tr>
<td>Rated speed</td>
<td>$N_{Load}$</td>
<td>2490 rpm</td>
</tr>
<tr>
<td>Inertia</td>
<td>$J_{Load}$</td>
<td>0.018 kgm²</td>
</tr>
</tbody>
</table>

V. SIMULATION RESULTS

Simulations are carried out with a two-inertia model by means of MATLAB/Simulink. Friction, backlash and other nonlinearities are neglected. The inner current control loop is approximated as a first-order time delay element with time constant $T_E = 250 \mu s$. The control input is limited to the rated torque of the drive motor $M_{max} = 36 \text{ Nm}$. Anti-wind up mechanism is implemented to the PI feedback controller to overcome saturation effects. Parameters of the two-mass model are presented in Table I.

Figure 5 shows simulation results with FBC. Transition time $T$ is set to 0.185 s to drive the system from zero speed to 20% of the nominal speed. Parameter $\gamma$ of the Gevrey function is set to 0.35 to avoid limitation by means of the converter. To clarify the flatness based feedforward control the feedback stabilizing controller is disabled in this case. It can be noticed that the flatness based feedforward control is able to force the nominal system to the defined reference trajectory.

VI. EXPERIMENTAL RESULTS

Measurements were taken on a test bench with a 5.5 kW converter fed induction motor, which is connected by a long driveshaft to a 6.4 kW converter fed servo induction machine which can induce a disturbance torque. Incremental encoders with 5000 pulses/revolution on motor side and 10000 pulses/revolution on load side are used for speed measurement. Parameters of the drive and load machines are presented in Table II.

A flywheel on the load side is used to increase the load inertia. The shaft torque is measured by a torque sensor with a strain gauge. The long shaft consists of aluminum with a quill shaft and a length of 1500 mm.

The flatness based speed control is implemented superimposed to a conventional field-oriented current control (FOC) structure oriented on the rotor-flux [25]. The control algorithm is implemented on a dSPACE DS1103 PPC controller board. The sampling frequency is set to 8 kHz.

Fig. 6 presents measurement results with flatness based feedforward control without feedback stabilizing controller. It can be seen that the feedforward control forces the system to the desired output trajectory close to the desired final value. Unconsidered friction leads to a deviation between the desired value and the actual value and to decrease of the drive speed. As mentioned before, a feedback stabilizing controller has to be added in order to compensate model uncertainties and external disturbances.

Therefore, the feedback stabilizing PI-controller is inserted for the following measurement results. The following results show the system variables of a step and disturbance response with PI-controller only (without feedforward control), Fig. 7. The reference value changes from 0% to 20% of the nominal speed at 0.1 s. A load torque with 60% of the nominal torque is induced from the load side at $t = 0.5 \text{ s}$. PI-control with reference step yields to mechanical oscillations as can be seen in the motor speed, load speed and shaft torque. Settling time amounts to $t_s = 0.215 \text{ s}$ due to the definition of the settling time with 2% error. The percent overshoot (PO) is 6.46 %. The maximum occurring shaft torque is twice as large as the rated motor torque $M_{S,max} = 73 \text{ Nm}$. Fig. 8 shows measurement results with PI-controller with the predefined reference trajectory instead of a reference step. It can be seen that no oscillations are excited with the trajectory as reference input. Consequently the maximum shaft torque is much smaller $M_{S,max} = 34 \text{ Nm}$. However, the settling time $t_s = 0.239 \text{ s}$ is higher than with a step as reference input. The percent overshoot $PO = 5.79 \%$ is in the same range.

Fig. 9 shows results with FBC. It can be noticed that motor and load speed follow the reference trajectory almost exactly. The percent overshoot $PO = 1.65 \%$ and the setting time $t_s = 0.177 \text{ s}$ are significantly smaller. The maximum shaft torque is also reduced to $M_{S,max} = 30 \text{ Nm}$.

In Fig. 10 a comparison of FBC with PI-control both with reference step and reference trajectory is shown. It can be seen that the FBC method shows the best results concerning overshoot, oscillations, maximum torque and settling time. The disturbance reaction shows no differences because of identical PI control parameters.

VII. CONCLUSION

A flatness based control approach is applied to the speed control of drive systems with elastically coupled loads. The drive system consists of a converter fed, current controlled induction machine. The transformation
of the conventional two-mass resonant model into controller form is presented for the derivation of the flatness based feedforward control. Gevrey functions are presented which are used as reference trajectories. System actuating value limitations are taken into account in the design of the reference trajectories.

Simulation and measurement results are presented and compared to conventional PI-control. Simulations have shown that the flatness based feedforward control is able to force the nominal system to the reference trajectory without feedback control. Nevertheless, a feedback controller is inserted due to disturbances and model uncertainties. Therefore, only the motor speed has to be measured to control the drive systems speed.

FBC is compared to a conventional PI-controller with both reference step and reference trajectory. The PI-control parameters are tuned appropriate with restricted pole placement to identical damping coefficients. Mechanical oscillations occur with conventional PI-control and a step as reference value. These oscillations can be suppressed with reference trajectories instead of reference steps. However, settling time increases significantly. FBC
shows excellent results concerning suppression of mechanical oscillations, overshoot and settling time. Thus, the stress of the system is reduced considerably. Due to the decoupling of reference reaction and disturbance reaction, the parameters of the feedback controller could be tuned only for disturbance reaction with FBC. Therefore, also a faster rejection of disturbances might be possible.

ACKNOWLEDGMENT
This work was funded by the Deutsche Forschungsgemeinschaft (German Research Foundation).

REFERENCES


