Comparative Study of Conventional PI-Control, PI-based State Space Control and Model Based Predictive Control for Drive Systems with Elastic Coupling

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Abstract—Three different control methods for the speed control of drive systems with elastically coupled loads are presented and compared. In drive applications where the load is connected to the driving motor with a drive shaft that has a finite stiffness, unwanted mechanical dynamics can occur. These unwanted dynamics can stress both mechanical and electrical drive components. Further, the shaft torsion, if neglected in the control synthesis, can dramatically reduce the achievable control performance. To overcome these challenges, the design, analysis, and a comparative study of three speed control methods for a drive system with resonant loads are carried out. The control methods considered are: a conventional PI-control, a PI-based state space control, and a model based predictive control. To ensure a suitable basis for their comparison, the three different speed control methods are designed with equal bandwidths and verified with the same test setup. Further, all speed control methods presented use only drive side speed measurement to control the drive speed.

Index Terms—AC machine; Adjustable speed drive; Asynchronous motor; Control of drive; Vibrations; State feedback; Predictive control; Test bench.

I. INTRODUCTION

It is a matter of common interest to realize competitive, energy-saving, and, if necessary, dynamic drive solutions. The use of adjustable speed drives composed of induction machines and frequency converters in addition to the further development of machines, power electronic components, and micro processors fulfill these high requirements of modern drives.

A typical adjustable speed drive topology which is widespread in industrial applications [1] consists of an inverter-fed AC motor and a load. The load is connected to the motor via transmission elements which have a non ideal transmission behavior such as finite torsional stiffness. This finite stiffness can lead to unwanted torsional oscillations and can stress both the mechanical and electrical components of the drive system. These problems occur for example in electric vehicles [2], rolling mill [3], or windmill [4] applications. For high dynamic speed control of drive systems with elastic coupling, the non-ideal behavior has to be considered in the control synthesis.

Control structures with proportional-integral (PI) controllers are usually used in industry for speed control of drive systems with elastically coupled loads [5]. Various design methods for tuning the PI control to reduce mechanical oscillations have been reported in the literature, cf. [6] or [7]. However, the PI-based speed control (PI<sub>Sc</sub>) method without additional feedback of the mechanical system states provides only a constricted pole placement of the closed-loop speed control and is not able to damp torsional oscillations effectively [8].

One possibility for reducing mechanical oscillations is the feedback of additional system states. These additional system states have to be measured or estimated by observers. A systematic analysis of speed control with different additional feedbacks is presented in [9]. Nevertheless, the poles of the closed loop speed control and the resulting system dynamics cannot be set freely.

A promising approach for the suppression of mechanical vibrations is the feedback of all system states—state space (SS) control [10]. The state space control approach has been studied extensively in the academic research literature [11] and is also attracting growing interest for industrial applications [5]. The state space control method allows a free pole-placement of the closed-loop control and thus a theoretically free choice of the system dynamics. The disadvantage of the state space control is the high implementation effort during the design of the control parameters and, moreover the measurement of all system states or their reconstruction from measurable signals by observers. PI-based state space speed control (PI-SS<sub>Sc</sub>) for drive systems with elastic coupling has already been analyzed, e.g. in [8], [12], [13], [14], and [15].

Another possibility for damping torsional oscillations without feeding back additional system states (besides the measured drive side speed) is the model based predictive control method (MBPC). Lately, the model based predictive control approach has been studied intensively for power electronic applications [16] and especially for drive systems with elastic coupled loads [17]. In [18], the MBPC approach was proven to be suitable for drive applications in terms of the achievable control performance, robustness against model-mismatches, and unexpected dynamics, as well as in terms of its online computation burden. The MBPC speed control has already been used in [19] for a stiff drive system and in [17], [20], and [21] for an elastic coupled drive system.
The proposed control approaches are based on a deeper theoretical basis starting with the well known PI\textsubscript{SC} across the PI-SS\textsubscript{SC} up to the MBPC\textsubscript{SC}. Except for the PI\textsubscript{SC}, the scientific analysis and research of the considered speed control methods for a drive system with elastic coupling is not yet concluded and still of high practical and scientific interest. Different publications and approaches such as those presented in the previous paragraphs can be found for every proposed speed control method, but until now (as far as the authors know) no comparative study between these control approaches has been presented in the literature. The present paper presents a survey and a comparative study to facilitate the choice of control approach for controlling a drive system with elastic coupled loads. Based on the given mechanical system, the available sensor system, and the achievable computation power for the control implementation, this work determines which of the considered control approaches is the optimal choice in terms of the dynamic behavior for its drive systems speed control. Further, the theoretical and practical analysis will highlight that the choice of the optimal speed control approach is dependent on the complexity of the applied control structure and the desired control performance.

The control methods considered will be compared in terms of the achievable dynamic performance indices and the stress on the mechanical system during reference and load steps. Further, the control approaches are compared in terms of their stability properties and online-computation effort. The complexity of the control design process and the possibilities of influencing the dynamic speed control performance will be studied for each speed control approach separately.

To ensure a practical and suitable basis for the proposed comparative study, all speed control methods are tuned to have the same bandwidth. Furthermore, only the drive side speed is measured.

This analysis is structured as follows: in the next chapter the mechanical system description and the torsional load model will be introduced. The speed control approaches to be considered are summarized in the third chapter. Furthermore, that chapter will present a comparison of the control approaches based on the proposed theoretical analysis. The fourth chapter introduces the test-bench, the measurement equipment, the measurement results obtained, and a comparison of the measurement results. A summary of the achieved results based on the theoretical and practical analyses will be presented in the fifth chapter. A conclusion is given in the last chapter.

II. SYSTEM DESCRIPTION AND TORSIONAL LOAD MODEL

A typical drive system that contains a gear, a torsional shaft (elastic coupling), and an additional inertia is presented in Fig. 1. The driving motor is fed by a PWM inverter to create variable stator voltage and frequency. The motor is connected to a load via a gearbox, a long torsional drive shaft, and an additional inertia. Thus, the mechanical part of these drive systems consists of several inertias and transmission elements with finite stiffness.

![Fig. 1. Topology of the drive system](image)

For an appropriate control synthesis, a model of the system has to be determined. The consideration of all inertias and transmission elements of the mechanical system leads to a multi-mass-model and due to the high number of system states, to a complex control synthesis. However, this multi-mass drive system has one dominant mechanical resonant frequency at \( f_{res}=41.3 \) Hz and the next much more damped resonant frequency is at approximately 400 Hz. In this case, the model of the mechanical system can be reduced to a simple two-mass model without an appreciable influence on the resultant control performance. Hence, the multi-mass model of the mechanical part is reduced to a two-mass model with the method of Rivin and Di [22]. This reduction leads to a two-inertia model with a drive side inertia \( J_M \), a load side inertia \( J_L \), and an elasticity coefficient \( c_T \) that represents the torsional stiffness of the drive shaft. The block diagram of the mechanical two-inertia model is shown in Fig. 2. The elastic shaft is presented with torsional elasticity \( c_T \) and internal damping \( d_T \), whereas for the proposed approach the damping coefficient is estimated. \( M_M \), \( M_S \) and \( M_L \) are the electromagnetic torque of the motor, the shaft torque, and the load torque, respectively, which can be considered as an external disturbance. \( N_M \) and \( N_L \) represent the motor and load speed, respectively. Nonlinearities, such as backlash and friction, are neglected in this mechanical system model. The parameters of the mechanical part and the parameters of the reduced two-inertia model are specified in Table I.

![Fig. 2. Block diagram of the two-inertia oscillation model](image)

The theoretical mechanical resonance frequency of the reduced model can be expressed as in (1). Note that the theoretical resonance frequency of the reduced two-inertia model corresponds with the measured resonance frequency of the multi-mass system of \( f_{res}=41.3 \) Hz.

\[
f_{res} = \frac{1}{2\pi} \sqrt{\frac{\left(J_M + J_L \right) \cdot c_T}{J_M J_L}} = 40.1 \text{ Hz}
\] (1)
### TABLE I
PARAMETERS OF THE MECHANICAL PART

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive side inertias:</td>
<td>JM = JDrive + JClutch + JShaft,1</td>
<td>0.0338 kgm²</td>
</tr>
<tr>
<td>Drive motor</td>
<td>JDrive</td>
<td>0.0180 kgm²</td>
</tr>
<tr>
<td>Clutch</td>
<td>JClutch</td>
<td>0.0151 kgm²</td>
</tr>
<tr>
<td>1/2 Drive shaft</td>
<td>JShaft,1</td>
<td>0.0007 kgm²</td>
</tr>
<tr>
<td>Load side inertias:</td>
<td>JL = JLoad + JFW + JShaft,2</td>
<td>0.1287 kgm²</td>
</tr>
<tr>
<td>Load motor</td>
<td>JLoad</td>
<td>0.0180 kgm²</td>
</tr>
<tr>
<td>Flywheel</td>
<td>JFW</td>
<td>0.1100 kgm²</td>
</tr>
<tr>
<td>1/2 Drive shaft</td>
<td>JShaft,2</td>
<td>0.0007 kgm²</td>
</tr>
<tr>
<td>Drive shaft data:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torsional stiffness</td>
<td>ct</td>
<td>1700 Nm/rad</td>
</tr>
<tr>
<td>Internal damping</td>
<td>ds (estimated)</td>
<td>0.15 Nms/rad</td>
</tr>
</tbody>
</table>

The transfer function of the reduced mechanical two-inertia model is summarized in (2).

$$G(s) = \frac{N_M}{M_M} = \frac{1}{s(J_M + J_L)} \frac{1 + s \frac{d_T}{c_T} + s^2 \frac{J_L}{c_T}}{1 + s \frac{d_T}{c_T} + s^2 \frac{J_L}{c_T}} J_M + J_L (J_M + J_L)$$  \tag{2}

### III. PROPOSED SPEED CONTROL APPROACHES

The different speed control methods are implemented in a conventional field-oriented control (FOC) structure. The block-diagram of the rotor-flux field-oriented drive control (R-FOC) structure is presented in Fig. 3.

#### A. Conventional PI-Speed Control

The PI-controller is widespread for speed control in industry because of its simple implementation and tuning of the control parameters [23]. The design of the control parameters is typically done using standard optimization methods, e.g., the symmetric optimum criterion [23]. This method is based on the idea of finding a controller that makes the frequency response of the closed control loop as close to one as possible for low frequencies [24]. However, the default setting of symmetrical optimum is not appropriate for drive systems with elastic couplings [25]. The controller gain has to be reduced in order to avoid high stress on the mechanical parts. Therefore, the reset time $T_r = k_P / k_I = 0.053s$ of the controller is kept constant whereas the proportional gain is reduced. Simulations and measurements have shown that satisfactory results can be achieved with the PI controller parameters presented in (3).

$$G_{PI}(s) = \frac{M_M(s)}{N_M(s) - N_M(s)} = k_P + \frac{k_I}{s} = 7.36 + \frac{138.1}{s}$$  \tag{3}

This tuning of the PI control parameters leads to a bandwidth of 8.7 Hz. Of course, other control parameter settings are possible, e.g., those presented in [6] or [23]. The main disadvantage of the conventional PI-control method is the limited pole placement possibilities and thus a limited speed control dynamics. Fig. 4 shows the block diagram of the conventional PI-speed control. An anti-windup limitation network is implemented, but only suggested as a saturation block.

#### B. PI-based State Space Speed Control

The PI-based state space control method includes the feedback of all mechanical system states [15]. Fig. 5 shows the block diagram of the PI-based state space speed control with an additional observer for the shaft torque and the load speed. Thus, only motor side speed has to be measured for speed control. The advantage of state space control is a theoretically free pole placement of the closed-loop speed control, if there is no limitation of the actuating value. Therefore high dynamic and high damping of torsional oscillations can be achieved. The difficulty of state space control is the determination of the control parameters.

![Fig. 3. Block diagram of the proposed rotor-flux oriented drive control structure](image3.png)

![Fig. 4. Block diagram of the conventional PI-speed control](image4.png)

![Fig. 5. Block diagram of the PI-based state space speed control](image5.png)

![Fig. 6. GPC speed controller with actuating value limitation network](image6.png)
The design of the control parameters is done using pole placement of the closed-loop poles. Therefore, the characteristic polynomial \( Q(s) \) of the closed-loop system is needed. Considering the mechanical part as a two-inertia model (2) and the delay of the inner current control loop approximated as a first-order time-delay with time constant \( T_C \), the characteristic polynomial of the closed-loop \( Q(s) \) is given by (4), where the internal damping \( d_I \) is neglected [15].

\[
Q(s) = s^4 + s^3 \frac{1 - k_p}{T_C} + s^2 \frac{J_m k_p - k_i + c_T T_C + J_m c_T}{J_m T_C} + s^2 \frac{J_m k_p + J_m k_i - c_T k_i - (J_m + J_m) c_T}{J_m T_C} + s \frac{c_T k_p - c_T k_i + c_T k_i}{J_m T_C} + c_T k_i
\]

(4)

Furthermore, a desired polynomial \( P(s) \) with five zeros, which determine the poles \( p_{z0} \) to \( p_{z4} \) of the closed-loop control, is introduced in (5).

\[
P(s) = (s - p_{z0})(s - p_{z1})(s - p_{z2})(s - p_{z3})(s - p_{z4})
\]

(5)

The poles of the closed-loop control are characterized by the damping coefficients \( D_1 \) and \( D_2 \) and by the eigenfrequencies \( \omega_{z0}, \omega_{z1}, \) and \( \omega_{z2} \) (6).

\[
P_{zs} = \omega_{zs}
\]

\[
p_{z1:2} = \omega_{z1}(D_1 \pm j\sqrt{1 - D_1^2})
\]

\[
p_{z3:4} = \omega_{z2}(D_2 \pm j\sqrt{1 - D_2^2})
\]

(6)

Comparing the coefficients of the characteristic polynomial \( Q(s) \) with the coefficients of the desired polynomial \( P(s) \) leads to a system of equations. The control parameters can be obtained by solving these equations. Due to the length of the expressions, the equations of the control parameters are not shown explicitly. The control parameters \( k_p, k_i, k_j \ldots k_d \) are depend on the system parameters \( J_m, J_l, c_T, T_C \) and on the desired poles of the closed-loop control \( p_{z0} \) to \( p_{z4} \). Thus, the dynamics of the speed control can be adjusted with the damping coefficients \( D_1 \) and \( D_2 \) and by the eigenfrequencies \( \omega_{z0}, \omega_{z1}, \) and \( \omega_{z2} \).

An appropriate approach is to place all poles except the dominant pole pair far to the left side in the complex s-plane. In that case, the dynamic is primarily influenced by the dominant pole pair \( p_{z1:2} \). But the speed control becomes very sensitive to measurement noise if the poles are placed too far to the left. Consequently, a trade-off has to be found. Satisfactory results and a bandwidth of 8.7 Hz have been achieved with the parameters presented in (7).

\[
\omega_{z0} = 30 \text{ rad/s}
\]

\[
\omega_{z1} = 70 \text{ rad/s}
\]

\[
\omega_{z2} = 300 \text{ rad/s}
\]

\[
D_1 = 0.61
\]

\[
D_2 = 0.707
\]

(7)

These desired poles of the closed-loop control lead to the control parameters of the PI-SS_SC presented in (8).

\[
k_p = 0.47 \quad k_i = -2.64 \quad k_j = -0.49
\]

\[
k_i = 141.06 \quad k_2 = 0.10 \quad k_4 = 0.66
\]

(8)

A disturbance observer [26] is implemented for the reconstruction of the shaft torque and load speed. The disturbance observer consists of a well-known Luenberger observer which contains the mechanical two-mass model (2), whereas the internal damping \( d_I \) is neglected and the inner current control loop is approximated as a first-order time delay. The observer is extended to a disturbance model, whereas the disturbance is assumed to be constant. The disturbance observer is tuned using pole placement. All poles of the observer are placed twice as far to the left as those of the closed-loop speed control.

C. Generalized Predictive Speed Control

The underlying concept of MBPC (also known as long-range predictive control) has been known since the late seventies. Many publications and books about its basic theory as well as specific applications have been published by now. Furthermore, many different methods have been developed, enhanced, or mixed in the past forty years. This leads to a wide range of predictive control algorithms that could be used for the problem being addressed in this paper.

Based on the literature [27]–[29], the MBPC algorithm of generalized predictive control (GPC) [30], [31] is chosen to control the drive side speed of the drive system with elastic coupled load considered here.

A GPC algorithm is composed of three parts [29]: a prediction equation, a cost function, and a resultant control law. In (9), the plant model of the chosen GPC algorithm is summarized as an integrated controlled auto-regressive moving average (CARIMA) model. \( A \) describes the denominator polynomial of the open loop plant transfer function and \( B \) describes the numerator polynomial. The polynomial \( T \) can be treated as a design polynomial to filter higher-frequency disturbances caused by model mismatch [32]. Furthermore, \( u(t) \) describes the control sequence (here the \( q \)-component of the stator reference current \( i_d \)), \( y(t) \) describes the process output sequence (here the measured drive speed \( N_{d} \)), \( d(t) \) describes the process distortion, \( z^{-1} \) the backshift operator, and \( d \) the dead time of the system. The GPC prediction equation is shown in (11) where the polynomials \( E_k \) and \( F_k \) are obtained from a Diophantine equation, cf. [30].

\[
A(z^{-1}) y(t) = z^{-d} B(z^{-1}) u(t-1) + T(z^{-1}) \frac{e(t)}{\Delta}
\]

(9)

\[
\Delta = 1 - z^{-1}
\]

\[
j(k + |t|) = F_k(z^{-1}) y(t) + E_k(z^{-1}) B(z^{-1}) \Delta u(t + k |t|)
\]

(11)

\[
J(N_1, N_2, N_3) = \sum_{j=N_1}^{N_3} \sum_{j=N_1}^{N_3} \Delta \left[ j(t + j|t| - w(t + j|t|) \right]^2
\]

(12)
To derive a control law from the prediction equation (11) for the GPC algorithm a cost function has to be defined, cf. (12). From (12) it can be seen that the quadratic cost function takes the predicted control error (\(\hat{y}-w\)) into account as well as the control signal change \(\Delta u\) to reach the commanded value. In Table II, the associated GPC design parameters are summarized.

**Table II**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_t)</td>
<td>Minimum costing horizon</td>
</tr>
<tr>
<td>(N_p)</td>
<td>Maximum costing horizon</td>
</tr>
<tr>
<td>(N_k)</td>
<td>Prediction horizon</td>
</tr>
<tr>
<td>(N_c)</td>
<td>Control horizon</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Command-weighting sequence</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Control-weighting sequence</td>
</tr>
<tr>
<td>(T(z^{-1}))</td>
<td>Filter polynomial</td>
</tr>
</tbody>
</table>

As highlighted in Fig. 6, an actuating value limitation has to be taken into account to design a realizable GPC speed control for the drive system with torsional loads. To reduce the implementation and the online computation effort this constraint is implemented a posteriori to the speed control system in this work. In [33], the theoretical background to implementing an actuating value limitation a posteriori to the system in this work. In [33], the theoretical background to implementing an actuating value limitation a posteriori to the MBPC system is presented. The calculation of the free system response is dependent on the change of the actual actuating value \(\Delta u\). Therefore the proposed solution is to not only limit the actuating value but also limit the change of the actuating value needed to calculate the free system response. For the drive system under consideration here, the speed control actuating value is the \(q\)-component \(i_{sq}\) of the reference stator-current. Therefore, the a posteriori implementation of the actuating value limitation can be formulated as in (13). The resultant structure of the GPC speed control is emphasized in Fig. 6.

\[
\begin{align*}
\dot{i}_{sq}^* &= \left[\begin{array}{c}
i_{sq}^* \\
\Delta i_{sq}(t) = i_{sq}^*(t) - i_{sq}(t-1)
\end{array}\right], \text{if } i_{sq}^*(t) \in [-i_{sq,max}, i_{sq,max}] \\
\dot{i}_{sq}^* &= i_{sq,max} \\
\Delta i_{sq}(t) &= i_{sq}(t) - i_{sq}(t-1), \text{if } i_{sq}^*(t) > i_{sq,max}
\end{align*}
\]

To summarize the MBPC control design and to ensure a suitable bandwidth for the proposed comparative study the GPC design parameters are chosen as in (14).

\[
\begin{align*}
N_t &= d = 2 & N_p &= N_q = 600 & N_c &= 1 \\
\lambda(j) &= 10 & \delta(j) &= 0.4 & T(z^{-1}) &= (1 - 0.97z^{-1})
\end{align*}
\]

Further information and additional analysis on the proposed MBPC is presented in [34] and [35].

**D. Comparison of the speed control methods concerning theoretical analysis**

This section gives a brief comparison of the speed control methods presented in this paper, concerning the possibilities of the controller designs, the complexity of implementation and control parameter tuning, and their stability properties.

Concerning the possibilities of controller design, the PI control method has two control parameters, \(k_I\) and \(k_P\), and for feedback, only that of the motor speed. Consequently, the choice of the closed-loop poles and the achievable dynamics are restricted. The PI-based state space control method includes feedback of all mechanical system states and leads to a theoretically free placement of the closed-loop poles and an enhanced dynamic. The structure of the GPC is similar to the structure of the PI and includes only the feedback of the motor speed. Thus, the pole placement of the closed-loop poles is restricted, too. But in addition, the GPC contains a predictive calculation of the actuating signal based on the systems model. Thus, an improvement of the dynamic can be achieved.

Due to the fact that the PI controller consists only of two control parameters, this method is the easiest to implement and to design. Comparing the PI-based state space control method and the generalized predictive controller in terms of their complexity of implementation does not lead to a clear decision. The PI-SS control contains six control parameters and the disturbance observer. The GPC has six design parameters and a filter polynomial. The design of the PI-SS is primarily done with two parameters: the prediction horizon \(N_p\), the command-weighting sequence \(\delta\), and the control-weighting sequence \(\lambda\). Nevertheless, all control parameters of PI-SS and GPC have to be chosen adequately. Depending on the specific knowledge one might have, it might be easier for some users to implement a state space controller and for other users to implement a predictive controller. However, the complexity of implementation and tuning of the PI-SS and the GPC is much higher than that of a conventional PI-SC, as shown in section III, A, B, and C. Further information about the design of PI-SS and GPC can be found in [15], [34], and [35].

The stability analysis of these control methods is done by analyzing the Bode diagram of the open-loop control systems. Fig. 7 shows the open-loop frequency responses of the proposed speed control methods. They are determined using a simplified model containing the approximated inner current control loop and the mechanical two-mass model (2). This figure presents the magnitude and phase plots of the control methods with the nominal systems model. Gain margin \(g_m\) and phase margin \(\varphi_m\) are determined for these control methods and are summarized in Table III. It can be seen that all of these control methods have stable performances. The highest gain and phase margin are achieved with PI-SS followed by PI-SC. The least stability properties are obtained with GPC.
The robustness concerning parameter uncertainties is analyzed by varying the system parameters of the mechanical system $J_m$, $J_L$, and $c_T$, whereas controllers and observer are tuned for the nominal system. The ranges of the parameter variations are given in (15).

\[
J_m = J_m \pm 0.3 \cdot J_m \\
J_L = J_L \pm 0.5 \cdot J_L \\
c_T = c_T \pm 0.5 \cdot c_T
\]

(15)

Gain and phase margin have been determined for every parameter variation. The minimum values of amplitude and phase margin of the uncertain system are presented in Table III. It can be seen that all the control methods considered yield stable control loops for the uncertain system. PI-SSSC achieves the highest gain margin, whereas the PISC achieves the highest phase margin. GPCSC also yields a stable control but with less robustness than PISC and PI-SSSC.

### IV. Measurements

Measurements were taken for each control method under the same conditions, such as actuating value limitation and sampling frequency. For a good comparability of the speed control methods under consideration, all controllers were tuned to the same bandwidth of 8.7 Hz. Of course another basis for the comparative study could have been possible. Consequently, other results concerning large-signal-behavior, small-signal-behavior, and disturbance rejection might be possible. Furthermore, only the drive side speed measurement was used for all control methods to control the drive speed. The load-side speed $N_L$ and the shaft torque $M_S$ are measured to analyze the proposed speed control methods.
TABLE IV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive motor:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rated power</td>
<td>$P_{\text{Drive}}$</td>
<td>5.5 kW</td>
</tr>
<tr>
<td>Rated torque</td>
<td>$M_{\text{Drive}}$</td>
<td>36 Nm</td>
</tr>
<tr>
<td>Rated speed</td>
<td>$N_{\text{Drive}}$</td>
<td>1455 rpm</td>
</tr>
<tr>
<td>Load motor:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rated power</td>
<td>$P_{\text{Load}}$</td>
<td>6.4 kW</td>
</tr>
<tr>
<td>Rated torque</td>
<td>$M_{\text{Load}}$</td>
<td>24.6 Nm</td>
</tr>
<tr>
<td>Rated speed</td>
<td>$N_{\text{Load}}$</td>
<td>2490 rpm</td>
</tr>
</tbody>
</table>

A. Test-bench description

To verify the theoretical approach, measurement results are presented. The mechanical part of the test bench is shown in Fig. 10. The setup consists of a 5.5 kW converter fed induction motor and a converter fed servo induction machine, connected by a long drive shaft. The servo induction machine is used to induce a disturbance torque. The parameters of drive and load machine are presented in Table IV.

A flywheel is used on the load side to increase the ratio $R$ between load side inertia and drive side inertia to $R = J_L/J_M = 3.81$. The shaft torque is measured with a torque sensor to determine the shaft stress. An incremental encoder with 5000 pulses/revolution on motor side and a resolver with a precision of 0.17° are used for speed measurement.

The different speed control methods are implemented superimposed to a conventional field-oriented current control (FOC) structure oriented to the rotor flux. The control algorithms are implemented on a dSPACE DS1103 PPC controller board. The sampling frequency is set to 3 kHz for each control method.

TABLE V

<table>
<thead>
<tr>
<th>Speed Control (SC)</th>
<th>Drive Side Speed Performance</th>
<th>Load Speed</th>
<th>Current- and Torque Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$PO^%$ $t_r^{\text{ms}}$ $t_s^{\text{ms}}$</td>
<td>$PO^%$ $i_{\text{Load}}^{\text{ref}}$ $M_{\text{ Shaft}}$ $dM_{\text{Shaft}}/dt$</td>
<td></td>
</tr>
<tr>
<td>Small-signal behavior (Reference Step from 40 rad/s to 43 rad/s) $U_{\text{DC}}=565$ V, $\Psi_R=0.6$ Vs, $i_{\text{max}}=15$ A, $f_s=3$ kHz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI-SC*</td>
<td>1.7</td>
<td>2.8</td>
<td>14.1 28.6 11.3</td>
</tr>
<tr>
<td>PI-SSc**</td>
<td>0.4</td>
<td>2.1</td>
<td>10.0 17.0 2.1</td>
</tr>
<tr>
<td>GPCSc***</td>
<td>0.7</td>
<td>2.2</td>
<td>10.3 21.3 2.6</td>
</tr>
<tr>
<td>Large-signal behavior (Reference Step from 40 rad/s to 60 rad/s) $U_{\text{DC}}=565$ V, $\Psi_R=0.6$ Vs, $i_{\text{max}}=15$ A, $f_s=3$ kHz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI-SC*</td>
<td>3.6</td>
<td>4.7</td>
<td>14.3 42.5 12.5</td>
</tr>
<tr>
<td>PI-SSc**</td>
<td>0.4</td>
<td>1.8</td>
<td>14.3 42.3 9.4</td>
</tr>
<tr>
<td>GPCSc***</td>
<td>0.6</td>
<td>1.3</td>
<td>14.3 44.5 7.5</td>
</tr>
<tr>
<td>Disturbance Rejection (Load Step from 0 Nm to 10Nm) $U_{\text{DC}}=565$ V, $\Psi_R=0.6$ Vs, $i_{\text{max}}=15$ A, $f_s=3$ kHz</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI-SC*</td>
<td>2.5</td>
<td>3.0</td>
<td>7.5   14.8 2.0</td>
</tr>
<tr>
<td>PI-SSc**</td>
<td>1.6</td>
<td>2.4</td>
<td>9.4   18.5 1.4</td>
</tr>
<tr>
<td>GPCSc***</td>
<td>2.3</td>
<td>2.6</td>
<td>9.4   17.9 1.6</td>
</tr>
</tbody>
</table>

#Percent Overshoot / Undershoot; ##Rise Time; ###Settling Time; Definitions cf. [24] * $k_p=7.36$, $k_i=138.09$, ** $N_p=600$, $\lambda=10$, $\delta=0.4$, *** $\omega_{\text{pp}}=300$, $\omega_{\text{z}}=70$, $\omega_{\text{z}}=300$, $D_1=0.61$, $D_2=0.707$

FIG. 10. Mechanical part of the test bench in the laboratory

TABLE VI

<table>
<thead>
<tr>
<th>Speed Control (SC)</th>
<th>Sampling Frequency $f_s=1/T_s$ [kHz]</th>
<th>Calculation Time $t_{\text{calc}}$ [μs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI-SC</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>GPCSc</td>
<td>3</td>
<td>35 ($N_p=200$) 73 ($N_p=600$)</td>
</tr>
<tr>
<td>PI-SSc</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>
B. Comparison of measurement results

The measurement results of the different speed control methods are presented in Fig. 8, Fig. 9, and Fig. 11. Fig. 8 shows the results of a reference step from 40 rad/s to 43 rad/s (small-signal behavior – actuating value not in limitation), the results of a reference step from 40 rad/s to 60 rad/s (large-signal behavior – actuating value in limitation) are presented in Fig. 9, and Fig. 11 displays the results of a disturbance rejection during a load step from 0 Nm to 10 Nm. The achieved control performances are characterized by the percentage overshoot PO of drive and load speed, rise time tr, and settling time ts of the drive speed, and the stress of the drive shaft, i.e., the maximum occurring shaft torque Mₘ and the maximum occurring rate of change of the shaft torque dMₛ/dt. These values are summarized in Table V for each control method.

Looking at the small-signal behavior, it can be concluded that the PIₛₐₗ leads to a higher PO and a longer settling time than PI-SSₛₐₗ and GPCₛₐₗ. Furthermore, torsional oscillations are obtained with PIₛₐₗ. This leads to a significantly higher shaft torque and a much higher dMₛ/dt. Overshoot, settling time, and stress of the drive shaft are reduced considerably with the proposed GPCₛₐₗ. However, the PI-SSₛₐₗ achieves the best control performance for small signal behavior.

PIₛₐₗ leads to the worst control performance even for large-signal behavior. PI-SSₛₐₗ leads to the lowest overshoot, but GPCₛₐₗ to the shortest settling time. In terms of the stress of the drive shaft, both GPCₛₐₗ and PI-SSₛₐₗ achieve comparably good results.

Analyzing the disturbance rejection, it is easily seen that PI-SSₛₐₗ leads to a much faster rejection of disturbances than PIₛₐₗ or GPCₛₐₗ. Again, PIₛₐₗ yields the worst dynamic behavior.

Table VI gives an overview of the total calculation time for the proposed speed control approaches including all transformations, controllers, and PWM calculations. PIₛₐₗ requires the least calculation time, tₐₗₑₙₗ=13 µs. The calculation effort of PI-SSₛₐₗ is only slightly higher, tₐₗₑₙₗ=16 µs, whereas the calculation time of GPCₛₐₗ is significantly higher and dependent on the prediction horizon. A calculation time of tₐₗₑₙₗ=73 µs was achieved with a prediction horizon of Nₚ=600.

V. SUMMARY OF THE COMPARISON

The results obtained based on the theoretical and practical analyses are summarized in Table VII. The measurement results confirmed that the conventional PIₛₐₗ leads to a low control performance and high stress on the mechanical system. Significantly better results concerning dynamic behavior and shaft stress were achieved with the GPCₛₐₗ. The best results were obtained with PI-SSₛₐₗ due to theoretically free choice of the closed-loop poles. Both PIₛₐₗ and PI-SSₛₐₗ have high stability and robustness properties. The proposed GPCₛₐₗ is stable and robust with respect to the parameter uncertainties presented, but yields less robustness than PIₛₐₗ and PI-SSₛₐₗ. The limitation of the GPCₛₐₗ is the required online calculation time. Depending on the prediction horizon the GPCₛₐₗ needs much more processing power than PIₛₐₗ or PI-SSₛₐₗ.

VI. CONCLUSION

A detailed comparison of three different control methods for the speed control of drive systems with elastic coupling: conventional PI-control, a PI-based state space control, and a model based predictive control (generalized predictive control) is presented. The control methods considered in this paper were designed and compared in terms of their dynamic behavior, the stress of the drive shaft, and their stability as well as robustness properties. Furthermore, the online calculation time, the possibilities of controller design, and the complexity of implementation and tuning of each control method were compared. For comparability, all control methods were tuned to the same bandwidth of 8.7 Hz, and measurements were taken for each control method under the same boundary conditions.

It can be concluded that the PI-control method is well known and very easily to design and to implement. But only poor speed control performance can be achieved as a consequence of its constricted pole placement. The control performance can be increased significantly by the use of a model based control method—GPC. Additional observers are not thereby required. But processing power may be a problem. The best control performance can be achieved with the PI-SSₛₐₗ due to its free pole placement. But an observer for the estimation of non-measured states is required. The implementation effort and the complexity relating to the control synthesis of the PI-SSₛₐₗ and GPCₛₐₗ are much higher than that of the conventional PIₛₐₗ.

Finally, the user has to decide, depending on the required control performance and the available control hardware, which of the control methods presented in this paper is the most convenient one.
ACKNOWLEDGMENT
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REFERENCES