Abstract—Design and analysis of PI state space control for grid-connected PWM converters with LCL filters based on pole placement approach is addressed. State space control offers almost full controllability of system dynamics. However, pole placement design is difficult and usually requires much experience. In this paper a suitable pole placement strategy is proposed, that ensures fulfilling the requirements, which are commonly specified with respect to rise time, overshoot, and proper resonance damping. Controller parameter expressions are derived in terms of system parameters and specified poles and zeros. Hence straightforward controller tuning for a particular system setting is possible. Performance is analyzed by means of transfer function-based calculations, simulations with Matlab, and experimental tests.

I. INTRODUCTION

LCL filters are increasingly used in grid-connected PWM converters due to their good damping characteristic. Active resonance damping is often required since passive damping is inflexible and inefficient. The choice of active damping method is a matter of sensor signals available. Manifold approaches have been proposed in literature [1]–[7]. However, fulfilling the requirements of fast rise time, small overshoot, and proper resonance damping as well as being robust is a challenge, even more when the ratio of LCL resonance to control frequency is low. Many approaches with partial state feedback for the purpose of resonance damping or sensorless solutions are not suitable [8], [9]. In [4] a combination of deadbeat and state space control is studied. Complete state feedback offers almost full controllability. Hence, grid-connected PWM converters with low ratio of LCL resonance to control frequency can be controlled as well. Even though state space control is a powerful approach, from industrial point of view there are drawbacks: many sensors and difficult controller design. High number of sensors increases costs and reduces reliability. For this reason, estimators or observers could be used in order to reduce the number of sensors. This topic is addressed in [2], [10]. A partial state feedback controller is presented in [11]. In the paper, that is presented in the following, the controller design is emphasized.

Linear quadratic (LQ) optimal control and pole placement are the most common ways of designing state space controllers. The LQ controller minimizes a quadratic integral criteria [4], [10]. For this purpose weighting matrices need to be designed. A robust controller with guaranteed gain and phase margins is obtained. However, the control design results in iterative tuning of weighting matrices. In [4] empirical guidelines for their selection can be found. The parameter calculation requires numerical tools for solving the mathematical problem, e.g. solution of a riccati equation. Therefore, controller tuning and transfer between system settings are difficult. Automatic controller tuning without manual fine tuning for a particular system setting is often important in industrial applications. Pole placement offers the derivation of control parameter expressions in dependence of system parameters and specified closed-loop performance characteristics. The idea of pole placement is to specify the closed-loop performance in terms of pole and zero locations. The control parameters are tuned in order to achieve the specified locations. The parameter calculation is straightforward, even if quite complex for systems of higher order. However, proper specification of closed-loop poles and zeros is required.

The objective of this paper is the development of a PI state space current control system that can be tuned straightforward by supplying the most important system parameters, namely LCL filter elements and control frequency. For this purpose the pole placement approach in discrete domain is addressed. Derivation of controller parameter expressions in terms of system parameters and specified closed-loop characteristics is shown. The latter are closed-loop poles and zeros that need to be specified in order to meet the requirements of rise time, overshoot, and proper resonance damping. The proposed pole placement strategy for the LCL filter-based system makes effectively use of the knowledge of the pole and zero locations of an PI-controlled L filter system. These are well known since standard PI design rules, e.g. symmetrical or technical optimum, are available [12], [13]. The system is described in section II. Control design is extensively studied in IV using the discrete system model derived in section III. Section V shows a theoretical analysis based on transfer function calculations. Results of simulation studies and experimental tests are presented in sections VI and VII, respectively. Finally, a conclusion is given.
II. SYSTEM DESCRIPTION

The grid-connected PWM converters with LCL filter is shown in Fig. 1. In addition to the DC link voltage all LCL system states are measured. The line voltages are measured for the purpose of synchronizing the control with the line voltage. For the test set-up control is implemented on a dSpace board. The converter is loaded by an inverter-fed induction machine.

The basic control structure is shown in Fig. 2. The DC link voltage is controlled with a PI controller. Active and reactive converter currents are controlled in synchronous reference frame with the proposed state space approach that is designed and analyzed in this paper.

III. MODELING

For control design and performance analysis a system model is derived. Because the outer DC link voltage is beyond the scope of this paper, its dynamics and control are not shown in this paper. The reader is referred to [8]. Current control and its dynamic are emphasized in this paper. Applying Kirchhoff’s laws and transformation into rotating frame yield the continuous model of the LCL-filter in dq frame [8]:

\[
\begin{align*}
\frac{d}{dt} i_{L,dq} &= \frac{1}{L_{fg}} \cdot (u_{L,dq} - u - C_{f,dq}) - j \omega \cdot i_{L,dq} \\
\frac{d}{dt} i_{C,dq} &= \frac{1}{L_{fc}} \cdot (u - C_{f,dq} - u - C_{dq}) - j \omega \cdot i_{C,dq} \\
\frac{d}{dt} u_{C,dq} &= \frac{1}{C_{f}} \cdot (i - L_{dq} - i - C_{dq}) - j \omega \cdot u - C_{f,dq}
\end{align*}
\]

(1)

The resonance frequency of the LCL-filter can be calculated [14]:

\[
\omega_{Res} = \frac{1}{\sqrt{\frac{L_{fg} + L_{fc}}{C_{f}}} \cdot \frac{L_{fg}}{L_{fc}}} = 2 \pi \cdot f_{Res}
\]

(2)

For the control design the zero frequency is important as well:

\[
\omega_0 = \frac{1}{\sqrt{L_{fg} \cdot C_{f}}} = 2 \pi \cdot f_0
\]

(3)

For control design the coupling terms in (1) are neglected. Hence the control can be designed for one axis and used for both axes. Taking them into account leads to more complex controller design due to the higher system order. In addition, parasitic resistances are neglected for control design. The calculation effort during the design is reduced considerably. It is of more importance to incorporate the delay due to the sampling, calculation, and PWM. With the state vector defined as \( x^{LCL} = [i_C, i_{Cf}, u_{DC}]^T \) the simplified continuous state space representation of the LCL filter for one axis can be written as (indices d or q omitted):

\[
x^{LCL}(t) = A^{LCL} \cdot x^{LCL}(t) + b^{LCL} \cdot u_C(k)
\]

(4)

with the system and input matrices

\[
A^{LCL} = \begin{bmatrix} 0 & 0 & \frac{1}{L_{fc}} \\ 0 & 0 & -\omega_{Res}^2 \cdot C_{f} \\ 0 & \frac{1}{C_{f}} & 0 \end{bmatrix} ;
b^{LCL} = \begin{bmatrix} -\frac{1}{L_{fc}} \\ 0 \\ 0 \end{bmatrix}
\]

For control purpose only sampled data are available and the control implementation is discrete as well. Therefore the control design is performed in discrete domain. Hence the continuous model is discretized:

\[
x^{LCL}(k + 1) = A_d^{LCL} \cdot x^{LCL}(k) + b_d^{LCL} \cdot u_C(k)
\]

(5)

whereas the discrete system matrices can be calculated with

\[
A_d^{LCL} = e^{\left(A^{LCL} \cdot T_c\right)};
b_d^{LCL} = (A_d^{LCL} - I) \cdot (A^{LCL})^{-1} \cdot b^{LCL}
\]

(6)

\( I \) equals the unity matrix of proper dimension. The exponential matrix is called fundamental matrix and it is the inverse Laplace transform of \((sI - A^{LCL})^{-1})\). Its calculation yields:

\[
A_d = \begin{bmatrix} 1 & -\frac{1 - \cos(\omega_{Res} T_c)}{T_{fe} \cdot C_f \cdot \omega_{Res}} & \frac{\sin(\omega_{Res} T_c)}{T_{fe} \cdot \omega_{Res}} \\ 0 & \cos(\omega_{Res} T_c) & C_f \cdot \omega_{Res} \cdot \sin(\omega_{Res} T_c) \\ 0 & \frac{\sin(\omega_{Res} T_c)}{C_f \cdot \omega_{Res}} & \cos(\omega_{Res} T_c) \end{bmatrix}
\]

(8)

\[
b_d = -\frac{1}{\omega_{Res} \cdot L_{fe}} \begin{bmatrix} \frac{\omega_{Res} \cdot T_c}{T_{fe} \cdot C_f} & \frac{\sin(\omega_{Res} T_c)}{T_{fe} \cdot \omega_{Res}} \\ -\omega_{Res} \cdot \sin(\omega_{Res} T_c) & \frac{1 - \cos(\omega_{Res} T_c)}{C_f} \end{bmatrix}
\]

(9)

There is a delay from the reference voltage \( u_c^* \) to the converter output voltage \( u_C \) [15]:

\[
u_C(k + 1) = u_C^*(k)
\]

(10)

Hence the following state space representation of the whole system taking into account the filter dynamics and delay is
used for control design

\[
\begin{bmatrix}
    i_C(k + 1) \\
    i_{Cf}(k + 1) \\
    u_{Cf}(k + 1) \\
    x(k + 1)
\end{bmatrix} = A_d^{sys} \cdot x(k) + \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    1
\end{bmatrix} \cdot u_C^*(k)
\]

whereas

\[
A_d^{sys} = \begin{bmatrix}
    a_{11} & a_{12} & a_{13} & b_1 \\
    a_{21} & a_{22} & a_{13} & b_2 \\
    a_{31} & a_{32} & a_{13} & b_3 \\
    0 & 0 & 0 & 0
\end{bmatrix}
\]

with \(a_{ij}\) and \(b_i\) \((i, j \in \{1, 2, 3\})\) being the entries of \(A_d\) and \(b_d).\) The converter current \(i_C\) is selected as output:

\[
y = [1 \ 0 \ 0 \ 0] \cdot x = x^T \cdot x
\]

The performance analysis takes the parasitic resistances into account.

IV. STATE SPACE CONTROL DESIGN

Different state space approaches are presented in literature. Pure proportional state feedback yields steady state errors. For zero steady state error an integrator can be integrated into the loop [4]. Here, further improvement is obtained by replacing the integrator with a PI controller. By that the tracking performance is improved due to the additional zero of the PI part which can be used for slow system pole compensation. Fig. 3 shows the control structure. The state feedback vector is defined as \(k^T = [k_{iC} \ k_{iCf} \ k_uCf \ k_uC]\) and the PI control parameters are the proportional gain \(k_p\) and the integrator time constant \(T_i.\)

The control design procedure can be divided into two steps. First, specification of desired poles and zeros is required. This implies the translation of the closed-loop requirements into pole and zero location. The second step is the controller parameter calculation. Basically, a relation between system parameters, pole and zero locations, and controller parameters is required.

A. Pole placement

Nowadays, PI control in synchronous reference frame is state of the art in current control of electrical drive systems, e.g., grid-connected PWM converters with L filters or induction motor drives [12]. For these applications, closed-current-loop performance requirements are commonly related to rise time and overshoot during steps. Rise time is typically specified as a couple of control periods. The overshoot is often limited by the converter current rating. Especially, in high power applications it is more stringent to limit the overshoot. Standard controller design rules and criteria, like technical or symmetrical optimum [12] [13], are known and can be used for designing PI controller for L filter systems. For the purpose of generality, symmetrical optimum is used [13]. Special tuning dedicated to a certain application is possible as well. Anyway, the locations of closed-loop poles and zeros, that fulfill particular requirements with respect to overshoot and rise time, are known for the L filter-based system.

However, resonance damping is an additional demand in LCL filter applications, in steady state as well as during transients. The strategy proposed in this paper makes advantage of knowing the closed-loop poles and zeros for the L filter-based system. The idea is to place all poles and the zeros, except those of resonance, as for L filter control. In addition, the impact of resonance is eliminated by proper shift of the resonance poles.

1) L filter system with pure PI control: Fig. 4 (left) shows schematically a typical pole and zero locations of an L filter system. In the open-loop system, there is a pole close to \(z = 1,\) due to the inductor. The one-sample delay gives an additional pole at the origin. The PI controller has a pole at \(z = 1\) and a zero on the real axis. The PI zero position is a matter of tuning. The closed-loop pole zero map is also shown in Fig. 4 (left). On the real axis there is a pole close to the zero, it is the compensated pole. The complex conjugated pole pair is dominant. In general, a complex pole \(p_i\) is characterized by its damping factor \(D_i\) and its frequency \(f_i: \)

\[
p_i = e^{-D_i \cdot \frac{1}{2\pi}} \cdot e^{j \sqrt{1-D_i^2} \cdot \frac{1}{2\pi}}
\]

Here, real poles are characterized just by its real part. An L Precise pole and zero locations of the L filter system with PI control tuned with symmetrical optimum \((k_p = L/(a_{SO} \cdot \ T_c), \ T_I = a_{SO}^2 \cdot T_c, \ \ a_{SO} = 3)\) can be calculated. The locations of the PI zero \(z_{1L},\) compensated pole \(p_{1L},\) and the dominant pole pair \((p_{2L}, p_{3L})\) are:

\[
\begin{align*}
    z_{1L} &= 1 - 1/a_{SO}^2 = 8/9, \quad p_{1L} = 0.88 \cdot z_{1L} \\
    p_{2L} &= \sqrt{D_{2L} \cdot 0.9}, \quad f_{2L} = f_e/12, \quad p_{3L} = p_{2L}^*
\end{align*}
\]

2) LCL filter system: Fig. 4 (right) shows schematically the location of poles and zeros of an LCL filter. In comparison with 4 (left), the LCL open-loop system has additional poles and zeros on the unity circle due to the resonance. The angles are related to its frequencies \(\omega_{Res},\) and \(\omega_0,\) respectively. Note that parasitic losses of the filter elements yield slight damping of the resonance. Hence the resonance poles are inside the
system zeros are not affected by feedback [16].

distance should be kept in order to have a robustness margin. A certain required. Placing the LCL resonance near the LCL zeros yields a small residual, since they compensate each other. A certain distance should be kept in order to have a robustness margin. Note that system zeros are not affected by feedback [16].

From previous considerations, the desired locations of the PI zero $z_1$, compensated pole $p_1^d$, dominant pole pair $(p_2^d, p_3^d)$, and non-dominant pole pair $(p_4^d, p_5^d)$ are finally specified to:

\[
\begin{align*}
  &z_1^d = z_1 \quad ; \quad p_1^d = p_1 L \\
  &D_2^d = D_2 L \quad ; \quad D_4^d = 0.2 \\
  &f_2^d = f_2 L \quad ; \quad f_4^d = f_0
\end{align*}
\] (16)

Note that $(p_2^d, p_3^d)$ and $(p_4^d, p_5^d)$ are complex conjugate pole pairs, that is: $p_2^d = p_2^*$ and $p_4^d = p_4^*$. See (3) for definition of $f_0$ and (15) for the L filter poles and zeros.

B. Controller parameter calculation

Once the desired poles and zeros have been specified, derivation of appropriate control parameter is necessary. Basically, the closed-loop transfer function is written in terms of system and control parameters. From comparison with the transfer function in terms of the desired poles and zeros, the control parameters can be calculated.

The closed-loop transfer function can be written as

\[
G_c(z) = \frac{I_c(z)}{L_c(z)^*} = \frac{G_{PI}(z)G_{RD}(z)}{G_{PI}(z)G_{RD}(z) + 1}
\] (17)

with

\[
\begin{align*}
G_{RD}(z) &= \zeta^T \left( z I - \left[ A_d^{sys} - L_d^{sys} \zeta^T \right] \right) \\
G_{PI}(z) &= k_p \frac{z + (T_c/T_s - 1)}{z - 1}
\end{align*}
\] (18) (19)

See also Fig. 3. The closed-loop transfer function from (17) can be rewritten:

\[
G_c(z) = \frac{(z - z_1) \cdot (z - z_2) \cdot (z - z_3)}{a_5 z^5 + a_4 z^4 + a_3 z^3 + a_2 z^2 + a_1 z + a_0}
\] (20)

The coefficients $a_i$ $(i \in [0, 5])$ consist of system and control parameters.

With the desired closed-loop poles and PI zero specified, the desired closed-loop transfer function can be expressed as

\[
G_c^d(z) = \frac{k_0 \cdot (z - z_1) \cdot (z - z_2) \cdot (z - z_3)}{(z - p_1^d) \cdot (z - p_2^d) \cdot \ldots \cdot (z - p_5^d)}
\] (21)

In order to obtain unity gain for zero frequency $(z = 0)$, the factor $k_0$ is used. Thus, no steady state error occurs. For the purpose of comparison with (20) it is rewritten:

\[
G_c^d(z) = \frac{(z - z_1) \cdot (z - z_2) \cdot (z - z_3)}{a_5^d z^5 + a_4^d z^4 + a_3^d z^3 + a_2^d z^2 + a_1^d z + a_0^d}
\] (22)

The coefficients $a_i^d$ $(i \in [0, 5])$ consist of specified poles.

Expressions for controller parameters in terms of system parameters and desired pole and zero locations are obtained from comparison of (20) and (22). The PI integrator time constant that determines $z_1$, follows directly from the numerator

![Fig. 4. Schematics of pole and zero locations. Left: PI-controlled L filter system, right: PI state space-controlled LCL filter system with the proposed placement strategy.](image-url)
Comparison of the denominator and subsequent rearrangement yield all the other control parameters. For the purpose of this paper’s clarity, the results are shown in the appendix, see (24). Even though they are quite long, a direct relation of the control parameters to the system parameters and the specified pole and zero locations are obtained.

V. THEORETICAL ANALYSIS

The proposed pole placement strategy is analyzed by means of transfer function-based calculations. For this purpose, the closed-loop transfer function from (17) is evaluated, whereas the system parameters from Tab. I and the controller parameters obtained from (24) with the specified poles and zeros from (16) are used. The LCL filter settings are selected such as a typical range of filter parameters with rather low resonance frequencies compared to the control frequency is covered. These settings are difficult to handle with other active resonance damping approaches [8], [17]. Two LCL filter settings with different filter capacitors are used. Filter capacitor voltages are filtered in order to reduce aliasing effects. The filter introduces a half-sample delay in the feedback signal. However, it is neglected for the analysis that follows.

Figs. 5 and 7 show the pole zero maps of the system with filter capacitor of 32.6 μF and 97.8 μF, respectively. Closed-loop pole differs slightly from the desired locations since the resistances are taken into account for the analysis. Note that they are neglected for controller parameter calculation. However, deviations are quite small for both filter settings.

Figs. 6 and 8 show step responses of the LCL filter as well as L filter system. The LCL system with higher resonance frequency response almost like the L filter system. The LCL system with lower resonance frequency shows that the additional oscillation due to the resonance decays slower. Thus the step response is slightly more oscillatory and the reference is crossed one time step.
later. Placing the resonance poles closer to the origin yields a faster decay, but as the distances to the resonance zero increase, the amplitude of the oscillation increases. Placing them closer to the zeros decreases the residual, but the decay time increases. In addition, a certain distance should be kept for robustness reasons. Note that the resonance frequency is in the same range as the closed-loop bandwidth, as can be seen from the frequency of the dominant poles and the resonance poles in Fig. 7.

It can be concluded that rise time and overshoot requirements as well as resonance damping are fulfilled. Note that the requirements are specified indirectly by choosing symmetrical optimum for design of an L filters PI controller. For low resonance frequencies its dampings is more difficult.

VI. Simulation Results
Performance is verified with Matlab/Simulink simulations. Switching of IGBTs is taken into account by using the toolbox PLECS. System and control parameters from the theoretical analysis are used. The purpose is to point out clearly the current control performance. Hence, the DC link capacitor is replaced with a constant DC voltage source and the outer DC link voltage loop is disabled for simulation study. Thus, current references can be chosen almost arbitrarily. The active current is controlled constantly to 30 A and the reactive current reference is varied for analysis of transients. Currents are sampled during zero space vectors at the beginning and the middle of the switching period. The filter capacitor voltage is oversampled with two times the control frequency and the average is used.

Fig. 9 shows the simulated step response which well confirms the calculated one from Fig. 6. Due to the couplings that have previously been neglected, the active current is slightly affected during the step as well. The line currents are more oscillatory during the step.

Fig. 10 shows the measured step response. It matches well with the simulation result from Fig. 9. However, due to grid voltage harmonics, low frequency distortions are visible in the currents even tough harmonic compensators are used. In simulations ideal line voltages are assumed. Fig. 11 shows the steady state current spectra. The resonance is well damped and the low frequency distortions are visible.

VII. Measurement Results
Performance is experimentally verified on a self-built 22 kW test drive in a laboratory environment. The grid-connected converter is loaded by an inverter-fed induction machine that is field-oriently controlled. Settings and operating points from theoretical studies and simulations are used. For this purpose the load torque on the motor shaft is adjusted in order to have a constant active power at 1000 rpm. The reactive current reference is varied for test purposes. The DC link voltage is controlled with PI control and slowly tuned resonant harmonic compensators are used for harmonics reduction. Feedback signals are sampled as described in the previous section.

Fig. 10 shows the measured step response. It matches well with the simulation result from Fig. 9. However, due to grid voltage harmonics, low frequency distortions are visible in the currents even tough harmonic compensators are used. In simulations ideal line voltages are assumed. Fig. 11 shows the steady state current spectra. The resonance is well damped and the low frequency distortions are visible.
VIII. CONCLUSION

A PI state space current control system for grid-connected PWM converters with LCL filters is developed and analyzed. Derivation of controller parameter expressions in terms of system parameters and specified closed-loop poles and zeros is shown. A pole placement strategy in discrete domain is proposed for the LCL filter-based system, that uses the pole and zero locations of a PI-controlled L filter system and suitable damping of resonance poles. Control performance is analyzed for two different LCL filter settings with different resonance frequencies.

The paper shows that the controller can be tuned straightforward using the derived expressions. Only the LCL parameters and control frequency are required. Good performance is proven for different system settings in comparison with an L filter system. High bandwidth with proper resonance damping and specified overshoot is obtained. Even for LCL filter systems with low resonance frequencies relative to the control frequency, the requirements are fulfilled, with only slightly more oscillating behavior during transients. Distortions of the feedback signals due to the converter switching can be avoided by proper filtering of measured filter capacitor voltage. Simulation and experimental results verify the theoretical results.

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APPENDIX

Controller parameter expressions are shown in (24) $\omega_R = \omega_{Res}, k_I = 1/T_I$.

$$k_{IC} = \frac{1}{4T_c} \frac{L_{fc}}{T_c \sin(w_R * T_c) (w_R^2 C_f L_{fc} \cos(w_R T_c) - \cos(w_R T_c) - w_R^2 C_f L_{fc} + 1)}$$

$$\left(\frac{w_R^2 T_c L_{fc} - 2 w_R T_c \cos(w_R T_c) + 2 w_R^2 T_c C_f L_{fc} \cos(w_R T_c) + 3 \sin(w_R T_c) - w_R T_c}{2 w_R T_c \cos(w_R T_c) + w_R T_c + 5 \sin(w_R T_c)}\right) \left((-p_1^d - p_2^d - p_3^d - p_4^d + 2 \cos(w_R T_c))\right)$$

$$k_{IC} = \frac{1}{4T_c} \frac{L_{fc}}{w_R C_f L_{fc}} \left(w_R^2 C_f L_{fc} \cos(w_R T_c) + w_R^2 \cos(w_R T_c) C_f L_{fc} + w_R^2 C_f L_{fc} - 1 + 2 \cos(w_R T_c) - \cos(w_R T_c)^2\right)$$

$$\left(w_R^2 T_c C_f L_{fc} + w_R T_c + \sin(w_R T_c) \right) \left((-p_1^d - p_2^d + p_3^d + p_4^d) + (p_1^d + p_2^d + p_3^d) - \cos(w_R T_c)\right)$$

$$2 w_R \cos(w_R T_c) + w_R T_c + \sin(w_R T_c) \right) \left((-p_1^d + p_2^d + p_3^d + p_4^d) + 2 \cos(w_R T_c)\right)$$

$$k_p = \frac{1}{2} \frac{L_{fc}^2 C_f w_R^2}{(-1 + \cos(w_R T_c)) (-1 + w_R^2 C_f L_{fc} T_c^2) T_c^2} \left((-p_1^d p_2^d p_3^d p_4^d) + (p_1^d p_2^d p_3^d p_4^d) + p_1^d p_2^d p_3^d p_4^d + p_1^d p_2^d p_3^d p_4^d + p_1^d p_2^d p_3^d p_4^d\right)$$

$$\left((-p_1^d p_2^d p_3^d p_4^d) + (p_1^d p_2^d p_3^d p_4^d) + p_1^d p_2^d p_3^d p_4^d + p_1^d p_2^d p_3^d p_4^d + p_1^d p_2^d p_3^d p_4^d\right)$$

$$k_{IC} = \frac{C_f L_{fc}^2 w_R^2}{2T_c^2 k_I \cdot (w_R^2 C_f L_{fc} \cdot \cos(w_R T_c) - \cos(w_R T_c) - w_R^2 C_f L_{fc} + 1)}$$

$$\left(1 + T_c k_I \right) \left((-p_1^d p_2^d p_3^d p_4^d) + (p_1^d p_2^d p_3^d p_4^d) + (p_1^d p_2^d p_3^d p_4^d) - \cos(w_R T_c)\right)$$

$$2 w_R \cos(w_R T_c) + w_R T_c + \sin(w_R T_c) \right) \left((-p_1^d + p_2^d + p_3^d + p_4^d) + 2 \cos(w_R T_c)\right)$$

$$k_{IC} = +2 \cos(w_R T_c) + 2 - p_1^d - p_2^d - p_3^d - p_4^d$$
REFERENCES


