1.6 What is a Potential

P is a potential:
This sentence implies many consequences for the function P:

- $\vec{x}$ describes the location (state)
- The value of P does not depend on the path (on the history of the system)
- The value only depends on the coordinates (the state):

$$P(\vec{x}_2) - P(\vec{x}_1) = \int_1^2 \vec{\nabla} P d\vec{x} \quad , \tag{1.11}$$

- dP is a total differential:

$$dP = \vec{\nabla} P d\vec{x} = \frac{\partial P}{\partial x_1} dx_1 + \frac{\partial P}{\partial x_2} dx_2 + \frac{\partial P}{\partial x_3} dx_3 \quad , \tag{1.12}$$

  i.e. the change of the coordinates defines completely the change of P.

- The "force" follows from the gradient

$$-\vec{\nabla} P \quad . \tag{1.13}$$

This relations we will illustrate for the example of an electric potential $W_e(\vec{x})$:

- $$\vec{E} = -\vec{\nabla} W_e(\vec{x}) \quad , \tag{1.14}$$

- $$W_e(\vec{x}_2) - W_e(\vec{x}_1) = -\int_1^2 \vec{E} d\vec{r} \quad , \tag{1.15}$$

- $$\begin{array}{ccc} dW_e & = & -\vec{E} d\vec{r} \\ \text{scalar} & & \text{vector} \\ \text{energy} & & \text{force} \end{array} \tag{1.16}$$

- **Consequences for measurements:**
  - Measure forces E_i in all directions x_i at each position $\vec{x}$
  - Calculate $dW_i = -E_i dx_i$ for the vector components along the path
  - The overall work is $dW = \sum_i dW_i$. This does not depend on the sequence of the measurements of the electric fields (forces).
  - This procedure needs a lot of single measurements for the three directions at each position along the way
- Direct measurement of the potential difference
  - just one measurement
- Both procedures are equivalent and you may switch between them, depending on which quantity is easier to measure.
- Just knowing that a function is a potential makes "life" much easier, even if you want to measure forces:
  - "Search for the easiest path from A to B on which you can sum up the forces to calculate the potential difference".