1.17 Calculation of the free energy of an ideal gas

We can calculate the Free Energy starting with the state function and the inner energy of an ideal gas

$$pV = NkT \qquad , \tag{1.35}$$

$$U = 3/2NkT \qquad . \tag{1.36}$$

(Note: In this notation U is not a potential, since T is not a coordinate of U!). Using Eq. (1.35) we get

$$p = -\frac{\partial F}{\partial V} = \frac{NkT}{V} \qquad , \tag{1.37}$$

i.e.

$$F(V,N,T) = -NkT(ln(V) + K(N,T)) \qquad . \tag{1.38}$$

The function $K(N,T)$ must still be calculated. Combining Eq. (1.36),

$$U = F + TS, \text{ and } S = -dF/dT \tag{1.39}$$

we find

$$\begin{aligned} \tfrac{3}{2}NkT \quad = \quad U \quad &= \quad -NkT\left(\ln(V) + K(N,T)\right) - T\left[-Nk\left(\ln(V) + K(N,T)\right) - NkT\tfrac{\partial K(T,N)}{\partial T}\right] \\ &= \quad NkTT\tfrac{\partial K(N,T)}{\partial T} \qquad . \end{aligned} \tag{1.40}$$

Consequently

$$K(N,T) = 3/2\ln(T) + K'(N) \qquad , \tag{1.41}$$

leading to

$$F(V,N,T) = -NkT\left[\ln(V) + 3/2\ln(T) + K'(N)\right] \qquad . \tag{1.42}$$

Successively integrating the state functions of a system allows to calculate the thermodynamic potential.
This procedure is necessary because in contrast to an electrical potential there is no way of measuring a thermodynamic potential directly. We therefor have to measure all "forces" in each state, determine thus the state functions which allow us to calculate the potential.