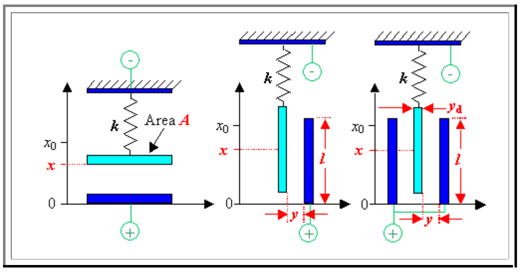
Solution to Exercise 7.1-2

Forces in Capacitive Structures

Here is the drawing once more to avoid jumping back and forth:



1. Find the proper relations for the forces pulling at the moveable plates for all three configurations.

- The total energy $\boldsymbol{E}$ for all three configurations is simply given by

$$E = \int U \cdot I \cdot \mathrm{d}t + \int k \cdot (x - x_0) \cdot \mathrm{d}x$$

- With the simple relations $C = Q/U$, $I = \mathrm{d}Q/\mathrm{d}t$ and therefore $U \cdot I \cdot \mathrm{d}t = (Q/C) \cdot (\mathrm{d}Q/\mathrm{d}t) \cdot \mathrm{d}t = Q/C \cdot \mathrm{d}Q$, we obtain

$$E = \frac{Q^2}{2C} + \frac{k \cdot (x - x_0)^2}{2}$$

$$= \frac{C \cdot U^2}{2} + \frac{k \cdot (x - x_0)^2}{2}$$

$$F = -\frac{U^2}{2} \cdot \frac{\mathrm{d}C}{\mathrm{d}x} - k \cdot (x - x_0)$$

- Without integration limits we cannot get proper signs (one energy term must decrease if the other one increases because we have energy conservation) - but that is not important here since we know that the spring force and the capacitive force must have opposite signs, and we are only interested in the capacitive force F_C.

For the capacity C and the force $F_C = ½U^2 \cdot (\mathrm{d}C/\mathrm{d}x)$ we obtain:

- **1.** Configuration:

$$C_1 = \frac{\epsilon_0 \cdot A}{x}$$

$$F_1 = \frac{U^2}{2} \cdot \frac{\epsilon_0 \cdot A}{x^2}$$

- **2.** Configuration:
x^* is the (easy to calculate) plate overlap for zero voltage. But since it disappears upon differentiation, we do not need to spell it

$$C_2 = \frac{\epsilon_0 \cdot (x^* - x) \cdot h}{y}$$

$$F_2 = \frac{U^2}{2} \cdot \frac{\epsilon_0 \cdot h}{y}$$

out.

- **3.** Configuration:
  C_3 is simply given by **2** C_2 in parallel

$$C_3 = 2C_2 = \frac{2\epsilon_0 \cdot (3l/2 - x) \cdot h}{y}$$

$$F_3 = 2F_3 = U^2 \cdot \frac{\epsilon_0 \cdot h}{y}$$

2. Compare the relative strength of the first and third configuration.

- If we simply take the relation F_1/F_3 for equal distances between the plates (i.e. $x = y$), we obtain

$$\frac{F_1}{F_3} = \frac{A}{y \cdot h}$$

- Considering that $y \cdot h << A$ for typical structures, configuration **1** can transmit much more force than the other ones for about identical size.

3. Discuss the pro and cons of the two configurations for driving an actual actuator.

In configuration **1** the force decreases with the square of the distance between the plates; in the extreme case of zero distance the plates would stick together *forever* (in reality a fuse will blow).

- The design rule is obvious: Use with extreme care!

In configuration **3** the force is independent on the position, which makes the design reasy. However, the force is relatively small.

- The consequences are obvious too: This is the preferred configuration, but you need to employ many combs to achieve sufficient force.