6.1.2 Light Amplification in Semiconductors

- If we produce a state of inversion, i.e. we have at least as many electrons in a high energy state as in the low energy state to which they fall be radiating recombinations, we will be able to amplify light.
  - Let's look at a real light amplifier now. The input light with an intensity I_0 enters the material, travels through the length of it, getting amplified all the time, and finally exits with a higher intensity I.
- For a quantitative analysis, let's consider a semiconductor which we keep in inversion conditions extending from $z = 0$ to $z = L$ along the z-axis. We now shine some light on it a $z = 0$ and with the spectral intensity $u_\nu(z = 0)$.
  - By definition, the rate for stimulated emission events, R_{se}, will increase in z-direction and so does the spectral intensity of the radiation, $u(\nu, z)$ which we now write somewhat simplified as $u_\nu(z)$.
  - We also assume that the inversion conditions are the same everywhere (i.e. they do not depend on z), and now define the *net rate of stimulated emissions* in short hand:

$$R^{net}{}_{se} = R_{se} - R_{fa} = R^{net}{}_{se}(z) =: R(z)$$

  - The interesting quantity, if we want to discuss the amplification of light, is $u_\nu(z)$, corresponding to the density of photons that, per second, travel along the z-axis. If we have any amplification at all, it will increase for increasing z and the rate of increase is somehow given by an amplification factor g_ν which must be a function of the "strength" of the inversion condition which in turn is tied to $R(z)$. We must expect that amplification is different at different frequencies and it is thus wise to index g with "ν". If g_ν is constant (which we can expect for constant R), the spectral intensity $u_\nu(z + \Delta z)$ at some point $z + \Delta z$ will be given by $u_\nu(z)$ as follows:

$$u_\nu(z + \Delta z) = u_\nu(z) + g_\nu \cdot u_\nu(z)\Delta z$$

  - This means that the intensity at the end of some length Δz of material is given by the intensity available at the entrance plus the part that is generated in the length increment considered. This part is proportional to the incremental length Δz available for amplification, the factor g_ν, which is defined by this equation and which will be properly called **gain coefficient**, and the intensity available.
- Making Δz arbitrarily small, i.e. moving from Δz to dz, yields a simple differential equation:

$$\frac{u_\nu(z + dz) - u_\nu(z)}{dz} = \frac{du_\nu(z)}{dz} = g_\nu \cdot u_\nu(z)$$

  - The solution, of course, is

$$u_\nu(z) = u_\nu(0) \cdot \exp(g_\nu \cdot z)$$

  - If we measure the intensity I of the light in some conventional units, we have the same relation, of course, because any measure of intensity at some frequency ν is always proportional to the number of photons. The increase in intensity then is

$$I_\nu(z) = I_\nu(0) \cdot \exp(g_\nu \cdot z)$$

- Formally, this is nothing but Beer's law of absorption if we introduce a *negative* absorption coefficient α, i.e $\alpha = -g_\nu$.
- While this was fairly straightforward, two questions remain:
  - *First*, an obvious question: What determines the gain coefficient?
  - *Second,* something a bit less obvious. At this point we have made all kinds of assumptions and approximations, and it is difficult to keep track of what kind of problem we are considering compared to reality. If you think about this, it all boils down to the following question: Are there *other losses* besides fundamental absorption to the photons and to the electrons in the inversion state that we have not yet included? Because if there are, we will have a harder time to amplify light than we think we have. Our present major goal, the amplification of light, then would be harder to achieve.
- Those are rather difficult questions which we will only consider summarily in this module. There are, however, links to more advanced stuff in what follows.

Gain Coefficient and Transparency Density

- Looking back at the simple example used for defining inversion, it is clear that the gain coefficient g_ν increases if the degree of inversion, i.e. the ratio of stimulated emission to fundamental absorption events, increases.
  - This implies that g_ν increases with increasing carrier density in the conduction band which in turn demands that $E_F{}^e$, the quasi Fermi energy of the electrons in the conduction band, moves deeper into the conduction band.
  - The gain coefficient g_ν , moreover, will be largest at the frequency corresponding to the energy levels where most electrons can be found. This level moves up in energy with increasing density of the electrons; for very few electrons it is of course E_C.
- Again, from the simple example used before, we can conclude that for the onset of inversion, i.e., for identical rates of fundamental absorption and stimulated emission, nothing happens in total: Exactly the same number of photons emerges at the output as fed into the input. We have

$$I_\nu(z) \;=\; I_\nu(0) \cdot \exp(g_\nu \cdot z) \;=\; I_\nu(0)$$

  - which demands $\exp(g_\nu \cdot z) = 1$ or $g_\nu = 0$.
  - The effect now will be that the semiconductor appears *completely transparent to the light*. The necessary density of electrons (for some fixed density of holes) is called **transparency density** $n^e{}_T$.
  - If the carrier density n^e increases beyond $n^e{}_T$, the maximum value of g_ν obtained at a certain frequency (which increases sightly with n^e) will increase, too, in a pretty much linear fashion. We find more or less empirically:

$$g_\nu{}^{max} \;=\; a \cdot (n^e - n^e{}_T)$$

  - The factor a may simply be considered to be a material constant called **differential gain factor**, since it depends more or less on material parameters like *density of states*, *band gap*, etc., and is hard to calculate for real materials. For example, GaAs has a value of $a \approx 2.4 \cdot 10^{-16}\ \mathrm{cm}^2$.
- In total, we have a complex dependence of g_ν on carrier density and frequency. An advanced module will give more details.

Additional Losses

- There are two kinds of possible losses that we may have to worry about.
  - We may loose some photons somehow which then cannot stimulate electrons to emit another photon.
  - We may loose electrons in some recombination channels; these electrons then aren't available for stimulated emission.
- The first kind of loss includes fundamental absorption, but that is already included in the theory so far. Are there other optical losses in the semiconductor?
  - Well, there are. Besides fundamental absorption, we also have the kind of absorption that prevents metals from being transparent: *Photons are generally absorbed by the free carriers*, i.e., by the electrons in the conduction band. If we increase the carrier density we will increase this effect.
- The second kind of loss includes all electrons that recombine through one of the other channels available: deep levels, direct recombinations, Auger recombination, excitons, ...
  - These recombination events not only reduce the number of available electrons, but the ones disapperaring via radiative recombination produce some light of their own – with the right wave length but with *random phases*, i.e., not coherent to the light we care for. This light also becomes amplified and induces a kind of **phase noise**.
  - Luckily, all these recombination losses are negligible for real devices.
- There might be more loss mechanisms, but we will sweep 'em all under the rug and simply combine everything there is (or might be) with respect to intrinsic losses (i.e., inside the semiconductor) in an **intrinsic loss coefficient** α_i.

- The decrease in intensity due to the intrinsic losses then is

$$I_{loss}(z) = I_\nu(0) \cdot \exp(-\alpha_i \cdot z)$$

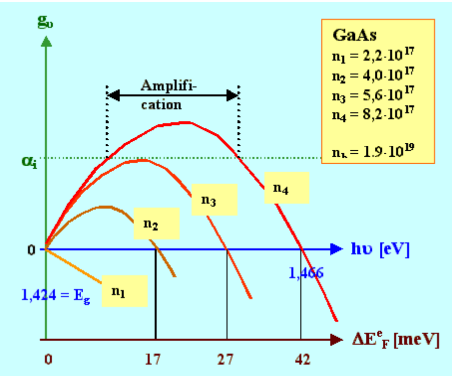
  - We can combine gain and losses then to the final equation linking the input to the ouptput:

$$I_\nu(z) = I_\nu(0) \cdot \exp[(g_\nu - \alpha_i) \cdot z]$$

  - For a constant gain coefficient along the crystal, the total output is then

$$I_\nu(L) = I_\nu(0) \cdot \exp[(g_\nu - \alpha_i) \cdot L]$$

- There is an important consequence from this equation: Amplifications demands that $g_\nu - \alpha_i > 0$ and that requires $g_\nu > \alpha_i$.
  - Just achieving inversion (corresponding to $g_\nu = 0$) thus is not good enough. There is a minimum or *threshold* value given by α_i before light amplification will occur. In other words: The density of electrons has to be larger than just the transparency density $n^e{}_T$.
- What this means in practice is shown below in a schematic way.
  - Shown are curves for g_ν for **GaAs** in a halfway realistic manner including some numbers.



  - A constant hole density of $n_h = 1 \cdot 10^{19}$ cm^{-3}, i.e., *heavily* doped **p**-type **GaAs** has been used as a reference. The electron density is raised by injection to four values marked n_1 through n_4 .
  - The gain coefficient g_ν is given as a function of the frequency (in terms of energy). A second scale shows the necessary level of the quasi Fermi energy for the electrons as $\Delta E^e{}_F$ above the conduction band edge.
  - For the electron density n_1 we have the onset of inversion. The gain coefficient is $g = 0$ for exactly one frequency corresponding to the band gap.
  - With increasing **n**, the gain coefficient is **> 0** for a portion of the frequency interval, peaking at about the frequency corresponding to $E_g + ½(\Delta E^e_F)$ - that's where most of the electrons are!
  - Only when g_ν is above the intrinsic loss coefficient α_i, which has been drawn in in a halfway realistic manner, some amplification occurs in the part of the spectrum indicated.
- More to this in the advanced module "gain coefficient"