Phase and Group Velocity

The velocity **v** of the electron in the free electron gas model is the *velocity with which the phase of the wave moves*, it is called **phase velocity v_P**.

- As long as a single plane wave symbolizes all that is to a particle, the *phase* velocity is automatically also the *particle* velocity **v**.

This is no longer true, however, for particles that are described as a *superposition* of (plane or other) waves with all kinds of wave vectors - and that means most *real* particles.

- While each individual wave of the set that now describes the particle still travels with its individual phase velocity v^i_P, the *maximum of the total amplitude of all waves* - signifying the most likely place to find the particle - travels with a velocity that must be identified with the particle velocity **v** and that is called **group velocity**.

- That the *group velocity* may be totally different from the *phase velocity* is nicely demonstrated by the example of standing waves, obtained by just combining two plane waves with wave vectors $\underline{k}$ and $-\underline{k}$. These waves have the same magnitude of the phase velocity, just opposite signs.

The result is a *standing wave* with maxima and minima that are fixed in space; we have group velocity $v_G = 0$. We may not really know where the particle is (locating it makes no sense under these coditions) but it certainly isn't going anywhere!

The group velocity for a particle still characterized by *one* wave vector $\underline{k}$ as. e.g. in a Bloch wave, is calculated by the simple formula

$$v_G = \frac{1}{\hbar} \cdot \frac{dE(\underline{k})}{d\underline{k}}$$

- i.e. it follows directly from the dispersion relation $E = E(\underline{k})$.

This distinction is essential to avoid confusion between the over-simplified quantum mechanical picture of the free electron gas, and the treatment of electrons (and holes) as particles later on.

- While the electron in the *free electron gas* would move through the lattice with a rather large velocity because group and phase velocity are identical, the "real" electrons are rather sluggish.

- This is of course a direct consequence of the fact that only electrons and holes near the band edges are important where the group velocities are low because $E(\underline{k}) \approx$ **constant**.

- *In other words:* The electrons we are concerned with resemble much more standing waves than fast moving plane waves. Or, in yet other words: They spent a lot of time hanging around the atoms before they move on to some other atom.