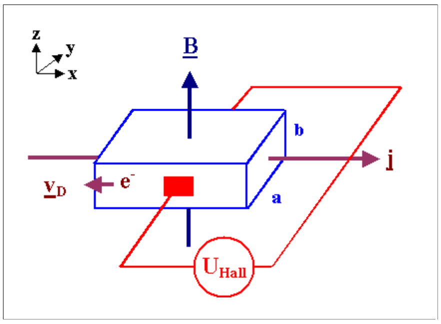
2.1.3 The Hall Effect

- This subchapter introduces *two* important topics: The **Hall effect** as an important observation in materials science and at the same time another irrefutable proof that classical physics just can't hack it when it comes to electrons in crystals.
  - The Hall effect describes what happens to current flowing through a conducting material - a metal, a semiconductor - if it is exposed to a magnetic field ***B***.
  - We will look at this in *classical* terms; again we will encounter a fundamental problem.
- The standard geometry for doing an experiment in its most simple form is as follows:



Relevant vectors; the current is carried by electrons

  - A magnetic field ***B*** is employed perpendicular to the current direction ***j***, as a consequence a *potential difference* (i.e. a *voltage*) develops at right angles to both vectors.
  - In other words: A **Hall voltage** U_{Hall} will be measured perpendicular to ***B*** and ***j***.
  - In yet other words: An electrical field $\underline{E}_{Hall}$ develops in ***y***-direction
  - That is already the essence of the Hall effect.

- It is relatively easy to calculate the magnitude of the *Hall voltage* U_{Hall} that is induced by the magnetic field ***B***.
  - First we note that we must also have an electrical field ***E*** parallel to ***j*** because it is the driving force for the current.
  - Second, we know that a magnetic field at right angles to a current causes a force on the moving carriers, the so-called **Lorentz force** F_L, that is given by

$$\underline{F}_L = q \cdot (\underline{v}_D \times \underline{B})$$

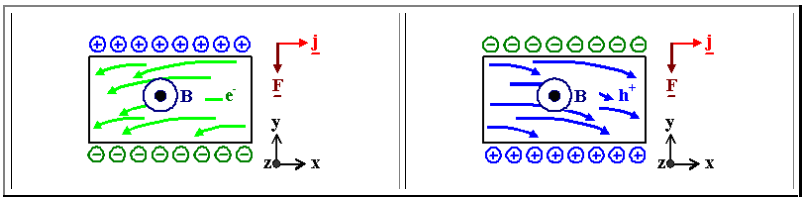
  - We have to take the drift velocity $\underline{v}_D$ of the carriers, because the other velocities (and the forces caused by these componentes) cancel to zero on average. The vector product assures that $\underline{F}_L$ is perpendicular to $\underline{v}_D$ and ***B***.
  - Note that instead the usual word "electron" the neutral term *carrier* is used, because in principle an electrical current could also be carried by charged particles other than electrons, e.g. positively charged ions. Remember a simple but important picture given before!
- For the geometry above, the Lorentz force $\underline{F}_L$ has only a component in ***y*** - direction and we can use a scalar equation for it. F_y is given by

$$F_y = -q \cdot v_{D,x} \cdot B_z$$

  - We have to be a bit careful: We know that the force is in ***y***-direction, but we do no longer know the sign. It changes if either q, v_D, or B_z changes direction and we have to be aware of that.
  - However, it is important to note that for a fixed current density j_x the direction of the Lorentz force is independent of the sign of the charge carriers – the sign of the charge and the sign of the drift velocity just cancel each other.
- With $v_D = \mu \cdot E$ and μ = mobility of the carriers, we obtain a rather simple equation for the force

$$F_y = -|q| \cdot \mu \cdot E_x \cdot B_z$$

- This means that the current of carriers will be deflected from a straight line in ***y***-direction. In other words, there is a component of the velocity in ***y***-direction and the surfaces perpendicular to the ***y***-direction will become charged as soon as the current (or the magnetic field) is switched on. The flow-lines of the carriers will look like this:



- The charging of the surfaces is unavoidable, because some of the carriers eventually will end up at the surface where they are "stuck".
- Notice that the sign of the charge for a given surface depends on the sign of the charge of the carriers. Negatively charged electrons (**e^-** in the picture) end up on the surface opposite to posively charged carriers (called **h^+** in the picture).
- Notice, too, that the direction of the force $\underline{F}_y$ is the same for both types of carriers, simply because both q and $\underline{v}_D$ change signs in the force formula

The surface charge then induces an electrical field E_y in y-direction which opposes the Lorentz force; it tries to move the carriers back.

- In *equilibrium*, the Lorentz force $\underline{F}_y$ and the force from the electrical field E_y in y-direction (which is of course simply $q \cdot E_y$) must be equal with opposite signs. We therefore obtain

$$-q \cdot E_y = -|q| \cdot \mu \cdot E_x \cdot B_z$$

$$E_y = \text{sgn}(q) \cdot \mu \cdot E_x \cdot B_z$$

The Hall voltage U_{Hall} now is simply the field in y-direction multiplied by the dimension d_y in y-direction.

- It is clear then that the (easily measured) Hall voltage is a *direct measure* of the mobility **μ** of the carriers involved, and that its **sign** or polarity will change if the sign of the charges flowing changes.

It is customary to define a **Hall coefficient** R_{Hall} for a given material.

- This can be done in different, but equivalent ways. In the link we look at a definition that is particularly suited for measurements. Here we use the following definition:

$$R_{Hall} = \frac{E_y}{B_z \cdot j_x}$$

In other words, we expect that the Hall voltage $E_y \cdot d_y$ (with d_y = dimension in y-direction) is proportional to the current(density) j and the magnetic field strength B, which are, after all, the main experimental parameters (besides the trivial dimensions of the specimen):

$$E_y = R_{Hall} \cdot B_z \cdot j_x$$

The Hall coefficient is a material parameter, indeed, because we will get different numbers for R_{Hall} if we do experiments with identical magnetic fields and current densities, but different materials. The Hall coefficient, as mentioned before, has interesting properties:

- R_{Hall} will change its sign, if the sign of the carriers is changed because then E_y changes its sign, too. It thus indicates in the most unambiguous way imaginable if positive or negative charges carry the current.
- R_{Hall} allows to obtain the mobility **μ** of the carriers, too, as we will see immediately.

R_{Hall} is easily calculated: Using the equation for E_y from above, and the basic equation $j_x = \sigma \cdot E_x$, we obtain for *negatively* charged carriers:

$$R_{Hall} = -\frac{\mu \cdot E_x \cdot B_z}{\sigma \cdot E_x \cdot B_z} = -\frac{\mu}{\sigma} = \frac{-\mu}{|q| \cdot n \cdot \mu} = \frac{-1}{|q| \cdot n}$$

The blue part corresponds to the derivation given in the link; n is (obviously) the carrier concentration.

- If one knows the *Hall coefficient* or the carrier concentration, the Hall effect can be used to measure magnetic field strengths B (not so easily done otherwise!).

Measurements of the Hall coefficient of materials with a *known* conductivity (something easily measurable) thus give us *directly* the mobility of the carriers responsible for the conductance.

- The *minus* sign above is obtained for *electrons*, i.e. *negative* charges.
- If positively charged carriers would be involved, the Hall constant would be *positive*.
- Note that while it is not always easy to measure the numerical value of the Hall voltage and thus of ***R*** with good precision, it is the easiest thing in the world to measure the *polarity* of a voltage.

Let's look at a few experimental data:

Material	**Li**	**Cu**	**Ag**	**Au**	**Al**	**Be**	**In**	Semiconductors (e.g. **Si**, G**e**, **GaAs**, **InP**,...)
R ($\times 10^{-24}$) cgs units	–1,89	–0,6	–1,0	–0,8	+1,136	+2,7	+1,774	*positive* or *negative* values, depending on "doping"

Comments:
1. the *positive* values for the metals were measured under somewhat special conditions (low temperatures; single crystals with special orientations), for other conditions negative values can be obtained, too.
2. The units are not important in the case, but multiplying with **$9 \cdot 10^{13}$** yields the value in **m^3/Coulomb**

Whichever way we look at this, one conclusion is unavoidable:

- In certain materials including *metals*, the particles carrying the electrical current are *positively charged* under certain conditions. And this is *positively not possible* in a classical model that knows only *negatively charged electrons* as carriers of electrical current in solids!

Again we are forced to conclude:

There is no way to describe conductivity in metals and semiconductors with *classical* physics!

Fragebogen / Questionaire
Multiple Choice Fragen zu 2.1.3