Waves

General Remarks

You may wonder why what follows is on the "Basic" level; it sure look rather scientifc. Well - if what follows is not basic to you, go out and protest about the shitty education you got. You just can't get more basic about how the universe ticks as in what follows

---

Basics

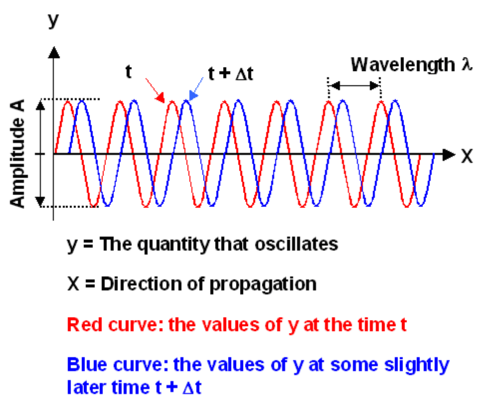
In the backbone, I gave a you a big thought: "*All that exists around us (including us) consists of atoms and photons*". The word "photon" is just another way of saying "electromagnetic radiation" which is just another way of saying "waves of something electromagnetic". The operating word here is "wave".

- Now for another big thought: Atoms consist of the elementary particles protons, neutrons and electrons, which are some kind of wave, too.

Everything, in a way, is a wave.

When, some time in the future, you hit your opponent with your Laser sword, you hit him with an electromagnetic wave. According to what we see on movies, a direct hit leaves lasting impressions. If you hit your opponent right now with the light beam of a present day (portable) Laser or flash light, he (or she) will be far less impressed than when you hit *him* with a steel sword. Mind you: never hit a Lady with your steel sword. If she consents to a ***tryst***, you use what, according to Freud, your sword signifies.
Sorry - I got distracted. The point is that equating hard steel (a bunch of atoms, after all) with a soft "wave" seems to be a bit ludicrous. Well, let's see.

What is a wave? The first association is always: something looking like this: . That's not too bad, but we need to go beyond that a bit. So let's make a nice drawing:



The definition of a (plane) wave

This kind of simple wave is called a called a **plane wave** because it would look the same in any plane at right angles to the direction of propagation.

- So something oscillates in time and, if it moves, in space. That quantity, for example, could be the height of some water relative to some fixed level. Now we have a wave propagating through water. At the same point in space the height goes up and down periodically, and along some line in the direction of propagation, the height goes up and down periodically, too.

The basic quantities needed to describe the kind of simple plane wave shown here are:

- **Amplitude A**: A measure of the "height" or better intensity of the wave.
- **Wavelength λ**: The distance between two neighboring maxima (or any neighboring identical values) of the wave amplitude.

- Propagation speed **$v = \Delta x/\Delta t$**: The velocity with which the wave (e.g. a maximum) moves in the propagation direction.
- **Cycle time T**. The time it takes to move exactly one wavelength. After a cycle time, everything looks exactly the same again. Obviously we have $T = v \cdot \lambda$.
- **Frequency $\nu = 1/T$**: How often per second a wave moved a distance λ, or how often per second everything repeats itself. Frequencies are measured in "cycles per second" (1/s) and they are always given in the unit "Hertz" (Hz) = 1/s
  Using ν instead of T gives the important relation $v = \nu \cdot \lambda$

Just for the hell of it, I give you the **basic equation** for describing a propagating plane wave. You can ignore it; it is not necessary for the understanding of what follows.

$$A(x, t) = A_0 \cdot \sin\left(2\pi \cdot \left(\frac{x}{\lambda} - \nu \cdot t\right)\right)$$

$$= A_0 \cdot \sin\left(\underline{k} \cdot \underline{r} - \omega \cdot t\right)$$

In three dimensions with vectors

$$= A_0 \cdot \exp\{i(\underline{k} \cdot \underline{r} - \omega \cdot t)\}$$

in complex notation

With $\underline{k}$ = "wave vector" = $2\pi/\lambda$; ω = circle frequency = $2\pi \cdot \nu$; $\underline{r}$ = position vector; $i^2 = -1$ = imaginary unit.

We can classify waves according to

- What "waves"?
- Wavelength.
- Frequency.
- Propagation speed.
- Energy transported.
- Interaction with other stuff.
- Kind of wave.

Let's go through that list point by point, always with some pertinent examples

What "waves"?

What "waves"? What kind of measurable physical quantity oscillates in time and space? Well - just about everything. Here is a list list of some quantities:

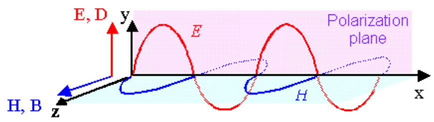
1. **Mechanical waves**. Some *coordinate* oscillates whenever we have mechanical waves. The mass on a spring goes up and down, the string of a musical instrument vibrates back and forth, as does the the membrane of a drum or the atoms in a crystal lattice. The amplitude is measured in meter.
2. **Sound waves** The *pressure* in a gas or liquid oscillates. It can do that in a solid, too - we just don't call it pressure anymore but stress or strain. Sound waves in air or in water belong into this category. The amplitude is measured in Pascal (Pa)
3. **Alternating current / voltage**. *Currents and voltages* inside electronic devices usually oscillate. Inside radios, TV's, quartz watches, mobile phones, computers, microwave ovens, generators - pretty much everything that is vaguely electronic - some electronic circuitry produces "alternating" voltages and / or currents. And no; I won't explain how it is done. The amplitude is measured in Volt (V) or Ampère (A).
4. **Electromagnetic waves**. Here it is the *electric and magnetic field strength* that oscillates. In contrast to everything above, this can happen in pure vacuum without any matter being present. If you feed current and voltage oscillations to a proper antenna, electromagnetic radiation is produced that can propagate through vacuum forever, and more or less well also through materials. The amplitude is given in terms of the electrical field strength Volt per meter (V/m) or magnetic field strength Tesla (T). You only need to know one; the other one can be calculated.
5. "**Matter waves**". The *wave function* ψ of a lonely particle or a big system of particles like the universe is the central quantity in quantum mechanics. It describes everything one can know about the system. It isn't called wave function for fun but because it usually contains "wavy" terms like those in the equations above. It has no unit, and its square gives the probability for finding particles at some chosen location.

- The list isn't complete but I covered the most important wavy things. And all of that is described by the equations above!

Wavelength, Frequency, Speed and Energy

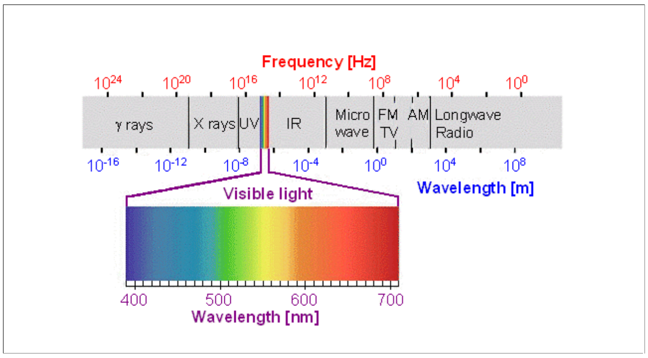
What kind of wavelengths do we encounter? Let's go through the list above:

- *Mechanical and Sound Waves*.
  Wavelength might be kilometers for those seismic waves produced when we have an earthquake. *Frequencies* then are very small and in the milli-Hertz (mHz) region.
  Sound, as we humans hear it, is in the frequency range **20 Hz - 20 kHz**. The speed of sound in air is about **330 m/s**, so wavelengths **$\lambda = v / \nu$** are in the **15 m - 15 cm** range. Frequencies higher than what humans can hear are called ultrasound. Bats do their thing around 100 kHz, and the medical ultrasound technique may go up to MHz. Sound waves transport very little energy. Even the 110 db noise from a chain saw delivers just about 0.1 W of acoustic power.
- *Current and voltage*
  Frequencies range from 16.66 Hz for railway power in Germany to "Tera" Hz (THz = 10^{12} Hz) in modern "full body" scanners in e.g. airports. While I'm strictly talking about voltages and currents inside some electronic "black box", it is pretty much impossible to keep the equipment form "leaking" electromagnetic radiation at high frequencies.
  Electrical signals inside materials like copper wires propagate with the speed of light ($\approx$ 300.000 km/s) divided by some numbers that account for the properties of the material and that are not too far from 1 for high frequencies. Typical propagation speeds then are about 2/3 of the speed of light.
  Wavelengths than range from 10.000 km for 15 Hz, via 200 m for 1 MHz, 20 cm for 1 GHz to 0,2 mm for 1 THz. As a rule of thumb, as soon as the wavelength is in the same general range as the size of your equipment, it will start to emit electromagnetic waves, i.e. act as a "sender".
- *Electromagnetic Waves*
  Visible light belongs to the group of electromagnetic waves, and so do radio waves, microwaves, X-rays and γ-rays; see below. What oscillates is a combination of an electrical field ***E*** (called electrical displacement ***D*** in materials) and a magnetic field ***H*** (called magnetic induction ***B*** in materials). The two fields are at right angles to each other and to the direction of propagation as shown below. That's why in this case we also can define a polarization as shown.



The "geometry" of an electromagnetic wave propagating in *x* direction.

Frequencies ν run from zero to as large as you like. In the higher frequency range quite a bit of energy ***En*** is transported according to ***En* = hν** (h = Planck's constant). Visible light carries energies around **2 eV**, enough to cause all kinds of chemical reactions like in photosynthesis. The eye or better brain perceives different frequencies or energies of visible light as color as shown below.
γ-rays can have energies of millions of eV, enough to cause havoc in whatever is in their way.
Wavelengths scale from very large to almost zero. Visible light is roughly around **0.5 µm** and that is the resolution limit for the smallest things one can see. Nothing smaller than about **0.5 µm** can be made visible in some microscope just using light.



Spectrum of electromagnetic waves

- **Wave function**
  In normal physics you know all about some particle with a mass and so on if you know where it is at some time. In other words you want to know the position vector $\underline{r}(t)$. In quantum mechanics you want to know something more abstract called the wave function $\psi(\underline{r}, t)$. It contains all the information about the particle, the trick is to ask the right question in a proper way.
  It may come as a surprise to you that the wave function of a solitary particle like an electron, proton or atom, moving in a straight line at some velocity **v**, has a wave function that is mathematically identical to that of a wave as given above: $\psi(\underline{r}, t) = \psi_0 \cdot \exp\{i(\underline{k} \cdot \underline{r} - \omega \cdot t)\}$.
  Wavelengths of electrons in a nice electron beam as we have it inside some old-fashioned TV picture tube or in electron microscopes are in the nm range or even below. That's why electron microscopes can "see" atoms.

Interaction with other Stuff

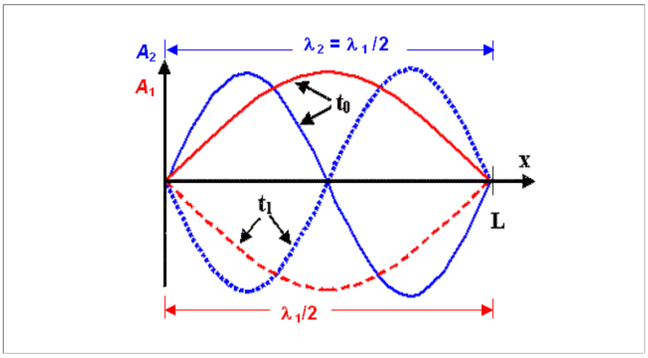
What happens if a wave hits matter? That's particular interesting for waves that can propagate in vacuum.

- First, however let's look at *mechanical waves* that can only exist in matter. Well, mechanical waves interact mechanically with other matter. A seismic wave will shake you, an accoustic wave wiggles your eardrum, and so on. Nothing special here.
- *Alternating currents and voltages* interact with "outside" materials that is not part of their own circuitry by sending alternating currents through whatever gets into contact. That might not be noticeable if the materials in contact is an insulator. But it also could produce plenty of power in an electrical motor, heat your steel to above the melting point, or serve justice the American way.
  If you contact some special piece of matter called "antenna" with sources of alternating currents and voltages, you produce an electromagnetic wave. If the frequencies are rather high (upper MHz and beyond) lots of things will act as antennae even if you don't want that.
- *Electromagnetic waves*, coming from vacuum (or air; not much different) do interesting things when they impinge on matter. But excluding very energetic γ-rays, they only and exclusively interact with *electrons*, usually the electrons inside atoms. That interaction can take many forms, and looking at that in some detail explains why microwaves heat food, lenses focus light, solar cells generate electricity, all hot objects glow, Lasers and LED's do what they do, UV light will tan you, plants grow, and so on and so forth.
- *Matter waves* as expressed by *wave functions* interact with other wave functions or electromagnetic waves (there is nothing else anymore) in a special and rather tricky way. In the end, however, most of what happens comes out exactly as you know it for all cases where the quantum nature of matter can be neglected.
  If that is *not* the case, strange things will happen. For example, particles like electrons can "tunnel" through otherwise impenetrable barriers, allowing us to make "Scanning Tunneling Microscopes" (STM) and many other useful devices.

Kinds of Wave

So far I only discussed propagating plane waves. There are, however, a lot of other kinds of waves. We have, for example, radial or spherical waves emanating from some point in all directions (e.g. the circular wave pattern you get when you pitch a stone into water), standing waves, e.g. inside organ pipes or your old recorder, very irregular waves like the pattern produced when you speak, or the "wave packets" of quantum physics.

- Here are some examples:



Standing Wave

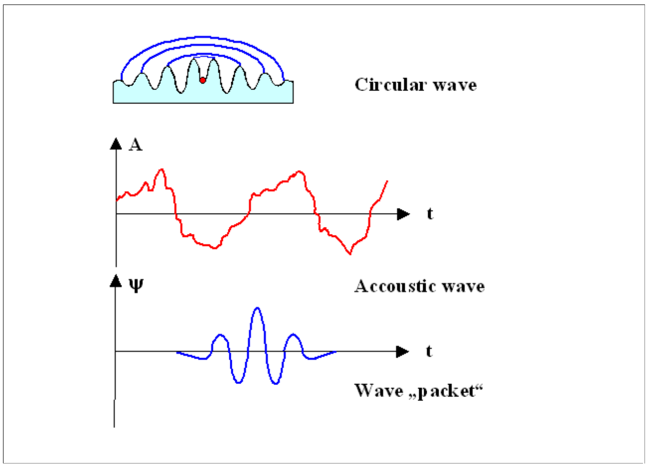
- Most important are the standing waves. They don't move in space; at any given point their amplitude just oscillates "in place" as shown above. Standing waves "live" as sound waves inside musical instruments but also as electron waves inside crystals.
  You can always construct a standing wave by superimposing two "running" plane waves that are identical except that they travel in opposite directions:

$$A_0 \cdot \exp\,\{i(\underline{k}\cdot\underline{r} - \omega\cdot t)\} \;\pm\; A_0 \cdot \exp\,\{i(-\underline{k}\cdot\underline{r} - \omega\cdot t)\} \;=\; \text{standing wave}$$

Why Math is so useful

- Even if you don't have a good idea about exponentials and complex numbers, it should become clear here that a bit of still rather elementary math is extremely useful. Describing with words everything contained in the equation above would take hours and at least two pages in any major newspaper feuilleton.

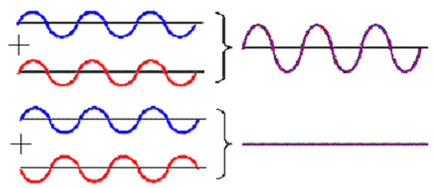
Let's look at a few more common wave types:



Types of waves

There are only two essentials to note about this

- **1.** *All* kinds of waves can be "made" by superimposing simple plane waves. This is also know as Fourier *synthesis*. Decomposing some given wave into its plane wave components is a Fourier *analysis*.
- **2.** Superposition produces the phenomenon of **interference** with extremely far-reaching consequences. Depending on their **phases** or the difference between the positions of the maxima, waves can double their amplitude or cancel each other completely as shown below.



The two extreme effects of interference: Amplitude doubling if the phases are identical, and cancellation if the phases are "opposite" or shifted by half a wavelength

- This is rather strange! Two waves, transporting energy and momentum, meet - and the result can be nothing!? Yes, that happens - in some part of space. Since energy and momentum must be conserved, something else now *must* happen in other parts of space.

Quantum theory assigns wave properties to all particles. Interference effects can now happen with everything. Old-fashioned classical particles could never, for example, cancel each other by interference - and this is why quantum effects appear to be so strange to classical people.

Science Link **Micros-copes**	One doesn't need to know all that much about waves to appreciate iron, steel and swords, and the technology going with that. However, if you admire all those microscopic pictures of steel structures and so on, you should at least know that all those microscopes rely very much on the wavy side of things, and all the interference effects going with it. And so does X-ray diffraction, the universal tool of the Material Scientist. The links supply details.	Science Link **X-Ray diffraction**