Solution to Exercise 4.3-2

▶ Given the type of lattice, the lattice constants of **Fe**, **Ni**, **Co** (look it up!), and the magnetization curves in chapter 4.3-2: How large are the magnetic moments of these atoms in terms of a Bohr magneton?

▶ Simple - but still a bit tricky.

   ● First we get the basic data:

- Lattice **Fe**: **bcc**; lattice constant **a = 2.86 Å**; atomic density $\rho_A(Fe) = 2/0.286^3$ atoms/nm^3 = **85.5 atoms/nm^3**
- Lattice **Nifcc**; lattice constant **a = 3.52 Å**; atomic density $\rho_A(Ni) = 4/0.352^3$ atoms/nm^3 = **91.7 atoms/nm^3**
- Lattice **Co**: **bcc**; lattice constant **a = 2.51 Å, c = 4.0 7Å**; atomic density $\rho_A(Co) = 2/[\frac{1}{2} \cdot c \cdot a^2 \cdot 3^{\frac{1}{2}}$ atoms/nm^3 = **90.1 atoms/nm^3**

   ● Then we realize that the curves in chapter 4.3-2 give the maximum magnetization, i.e. the magnetization state for all magnetic moments perfectly aligned. From the figure we can deduce the following numerical values for the saturation magnetization m_{Sat}:

- $m_{Sat}(Fe) = 17 \cdot 10^5$ **A/m**
- $m_{Sat}(Fe) = 5 \cdot 10^5$ **A/m**
- $m_{Sat}(Fe) = 14 \cdot 10^5$ **A/m**

   ● However, the units shown are **A/m**, which are not what we would expect. Obviously we must convert this to - well, what exactly?

▶ If we look at a Bohr magneton, m_{Bohr}, we have

$$m_{Bohr} \; = \; 9.27 \cdot 10^{-24} \text{ Am}^2$$

   ● Obviously, the unit we need is **Am^2**. We obtain that by multiplying the **A/m** by **m^3**, which makes clear that the m_{Sat} numbers given are per **m^3** - as they should be!

▶ The magnetic moments **m$_A$** per atom are thus

$$m_A \; = \; \frac{m_{Sat}}{\rho_A}$$

What we obtain is

$$m_A(Fe) = \frac{17 \cdot 10^5 \text{ A/m}}{85.5 \text{ atoms/nm}^3} = \frac{17 \cdot 10^5 \text{ A} \cdot 10^{-27} \text{ m}^3}{85.5 \quad \text{m}} = 1.98 \cdot 10^{-23} \text{ A/m}^2 = 2.14 \text{ m}_B$$

$$m_A(Ni) = \qquad\qquad 5.45 \cdot 10^{-24} \text{ A/m}^2 = 0.588 \text{ m}_B$$

$$m_A(Co) = \qquad\qquad 1.55 \cdot 10^{-23} \text{ A/m}^2 = 1.67 \text{ m}_B$$

▶ Now that is an interesting result! It's satisfying because we actually get sensible numbers close to a Bohr magneton, and it's challenging because those numbers are not very close to **1**, **2**, or possibly **3**.

   ● For example, how can a **Ni** atom have a magnetic moment of **0.588 m$_B$**, and a **Fe** atom one of **2.14 m$_B$**, considering that the spins of the electrons carry exactly **1 m$_B$**?

▶ There are two possibilities for this apparent discrepancy:

1. Our calculation is somehow a bit wrong
2. There are some effects not yet discussed that change the magnetic moment an atom in a crystal lattice carries around with itself somewhat.

   ● The first possibility can be ruled out, because in standard textbooks, e.g. in the "*Kittel*" we find the following values for **m$_A$**

- **m$_A$(Fe) = 2.22 m$_B$**
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- $m_A(Ni) = 0.606\ m_B$
- $m_A(Co) = 1.72\ m_B$

Not identical, but close enough. In fact, looking more closely, the Kittel values are for **T = 0 K**, whereas our values are for room temperature **T = 300 K** and thus should be a bit smaller.

● Obviously, this leaves us with some effects not yet discussed. What these effects could be, we can only guess at. Here is a short list:

- There might be some interaction between the spins of the electrons and the "orbits" of the electrons that modifies the magnetic moment
- The free electrons of the electron gas in our metal also "feel" the ordered spins of the atoms and react to some extent by adjusting their spins.

● This can lead to quite sizable effects. Dysprosium (**Dy**), for example, a rare earth metal, is a ferromagnet below its Curie temperature of **88 K** and its atoms than carry an $m_A(Dy) = 10.2 m_B$.