

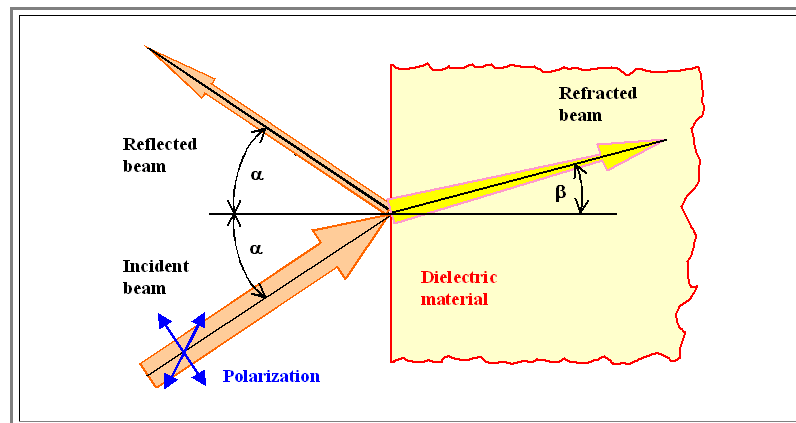
3.7 Dielectrics and Optics

3.7.1 Basics

- ▶ This subchapter can easily be turned into a whole lecture course, so it is impossible to derive all the interesting relations *and* to cover anything in depth. This subchapter therefore just tries to give a strong flavor of the topic.
- ▶ We know, *of course*, that the **index of refraction** n of a non-magnetic material is linked to the dielectric constant ϵ_r via a simple relation, which is a rather direct result of the [Maxwell equations](#).

$$n = (\epsilon_r)^{1/2}$$

- ▶ But in learning about the origin of the dielectric constant, we have progressed from a simple constant ϵ_r to a complex **dielectric function** with frequency dependent real and imaginary parts.
 - What happens to n then? How do we transfer the wealth of additional information contained in the *dielectric function* to optical properties, which are to a large degree encoded in the *index of refraction*?
- ▶ Well, you probably guessed it: We switch to a **complex index of refraction**!
 - But before we do this, let's ask ourselves what we actually want to find out. What are the optical properties that we like to know and that are *not* contained in a simple index of refraction?
 - Lets look at the *paradigmatic* experiment in optics and see what we *should* know, what we *already* know, and what we *do not* yet know.



- ▶ What we have is an electromagnetic wave, an *incident beam* (traveling in vacuum to keep things easy), which impinges on our dielectric material. As a result we obtain a *reflected beam* traveling in vacuum and a *refracted beam* which travels through the material. What do we know about the three beams?
 - The *incident beam* is characterized by its wavelength λ_i , its frequency ν_i and its velocity c_0 , the direction of its **polarization** in some coordinate system of our choice, and the arbitrary angle of incidence α . We know, it is hoped, the simple **dispersion relation** for vacuum.

$$c_0 = \nu_i \cdot \lambda_i$$

- c_0 is, of course, the velocity of light in vacuum, an absolute constant of nature.
- ▶ The *incident beam* also has a certain *amplitude* of the electric field (and of the magnetic field, of course) which we call E_0 . The *intensity* I_i of the light that the incident beams embodies, i.e. the energy flow, is proportional to E_0^2 - never mix up the two!
- ▶ The *reflected beam* follows one of the basic laws of optics, i.e. *angle of incidence = angle of emergence*, and its wavelength, frequency and magnitude of velocity are identical to that of the incident beam.
 - What we do *not know* is its *amplitude* and its *polarization*, and these two quantities must somehow depend on the properties of the incident beam *and* the properties of the dielectric.
- ▶ If we now consider the *refracted beam*, we know that it travels under an angle β , has the same frequency as the incident beam, but a wavelength λ_d and a velocity c that is different from λ_i and c_0 .
 - Moreover, we must expect that it is *damped* or *attenuated*, i.e. that its amplitude decreases as a function of penetration depth (this is indicated by decreasing thickness of the arrow above). All parameters of the refracted beam may depend on the polarization of the incident beam.
 - Again, *basic* optics teaches that there are some simple relations. We have

$\frac{\sin \alpha}{\sin \beta} = n$	Snellius law
$n = \frac{c_0}{c}$	From Maxwell equations
$c = v_i \cdot \lambda_d$	Always valid
$\lambda_d = \frac{1}{n} \cdot \lambda_i$	From the equations above

- A bit more involved is another basic relations coming from the Maxwell equations. It is the equation linking c , the speed of light in a material to the material "constants" ϵ_r and the corresponding magnetic permeability μ_0 of vacuum and μ_r of the material via

$$c = \frac{1}{(\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_r)^{1/2}}$$

- Since most optical materials are not magnetic, i.e. $\mu_r = 1$, we obtain for the index of refraction of a dielectric material our relation [from above](#).

$$n = \frac{c_0}{c} = \frac{(\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_r)^{1/2}}{(\mu_0 \cdot \epsilon_0)^{1/2}} = \epsilon_r^{1/2}$$

- Consult the [basic optics](#) module if you have problems so far.

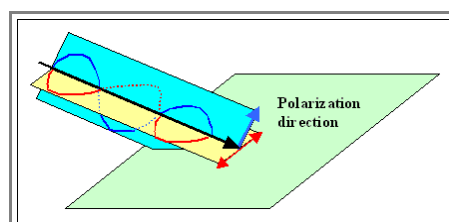
If we now look at not-so-basic optics, we encounter the **Fresnel laws** of diffraction.

- Essentially, the **Fresnel laws** give the **intensity of the reflected beam** as a function of the **angle of incidence**, the **polarization** of the incident beam, and the **index of refraction** of the material.

- The Fresnel laws are not particularly easy to obtain (consult the basic module [Fresnel laws](#)), but the results are easy.

First, we must distinguish between the two basic polarization cases possible:

- The incident light might be polarized in such a way that the vector of the electrical field $\mathbf{E}$ lies either **in** the plane of the material, or **perpendicular** to it, as shown below. Anything in between than can be decomposed into the two basic cases.



- Lets call the amplitudes of the reflected beam A_{para} for the case of the polarization being **parallel** to the plane (= surface of the dielectric), and A_{perp} for the case of the polarization being **perpendicular** to the plane (blue case) as shown above. For a unit amplitude of the incident beam, the Fresnel laws then state

$$A_{\text{para}} = - \frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)} \quad A_{\text{perp}} = - \frac{\tan(\alpha - \beta)}{\tan(\alpha + \beta)}$$

We can substitute the angle β by using the relation [from above](#) and the resulting equations then give the intensity of the reflected light as a function of the material parameter n . Possible, but the resulting equations are no longer simple.

- In order to stay simple and focus on the essentials, we will now consider only cases with about *perpendicular* incidence, i.e. $\alpha \approx 0^\circ$. This makes everything much easier.
- At *small* angles we may substitute the argument of the **sin** or **tan** for the full function, and obtain for *both* polarizations

$$A \approx - \frac{(\alpha - \beta)}{(\alpha + \beta)}$$

- Using the expression for n from above for small angles too, we obtain

$$n = \frac{\sin \alpha}{\sin \beta} \approx \frac{\alpha}{\beta}$$

- Now we keep in mind that we are usually interested in [intensities](#), and not in amplitudes. Putting everything together, we obtain for the reflectivity R , defined as the ratio of the intensity I_r of the reflected beam to the intensity I_i of the incident beam for almost perpendicular incidence

$$R = \frac{I_r}{I_i} = \frac{(n - 1)^2}{(n + 1)^2}$$

The grand total of all of this is that if we know n and some basics about optics, we can answer most, *but not all* of the questions [from above](#). But so far we also did not use a *complex* index of refraction either.

- In essence, what is missing is any statement about the *attenuation* of the refracted beam, the **damping of the light** inside the dielectric - it is simply not contained in the equations presented so far.
- This cannot be right. Electromagnetic radiation does not penetrate arbitrarily thick (and still perfect) dielectrics - it gets pretty dark, for example, in deep water even if it is perfectly clear.
- In not answering the "damping" question, we even raise a new question: If we include damping in the consideration of wave propagation inside a dielectric, does it change the simple equations given above?

The *bad* news is: It does! But relax: The *good* news is:

- All we have to do is to exchange the "simple" refractive index n by a *complex refractive index* n^* that is directly tied to the complex dielectric function, and everything is taken care of.
- We will see how this works in the next paragraph.