

Solution to Exercise 1.3-1

Derive and Discuss numbers for μ

First Task: Derive numbers for the mobility μ .

- First we need typical conductivities and electron densities in *metals*, which we can take from the [table in the link](#).
- At the same time we expand the table a bit

Material	ρ [$\Omega \text{ cm}$]	σ [$\Omega^{-1} \text{ cm}^{-1}$]	Density $d \times 10^3$ [kg m^{-3}]	Atomic weight w [$\times 1 \text{ u} = 1,66 \cdot 10^{-27} \text{ kg}$]	$n = d/w$ [m^{-3}]
Silver Ag	$1,6 \cdot 10^{-6}$	$6,2 \cdot 10^5$	10,49	107,9	$5,85 \cdot 10^{28}$
Copper Cu	$1,7 \cdot 10^{-6}$	$5,9 \cdot 10^5$	8,92	63,5	$8,46 \cdot 10^{28}$
Lead Pb	$21 \cdot 10^{-6}$	$4,8 \cdot 10^4$	11,34	207,2	$3,3 \cdot 10^{28}$

For the mobility μ we have [the equation](#)

$$\mu = \frac{\sigma}{q \cdot n}$$

With q = elementary charge = $1,60 \cdot 10^{-19} \text{ C}$ we obtain, for example for μ_{Ag}

$$\mu_{\text{Ag}} = \frac{6,2 \cdot 10^5}{1,6 \cdot 10^{-19} \cdot 5,85 \cdot 10^{28}} \frac{\text{m}^3}{\text{C} \cdot \Omega \cdot \text{cm}} = 66,2 \frac{\text{cm}^2}{\text{C} \cdot \Omega}$$

The unit is a bit strange, but remembering that $[\text{C}] = [\text{A} \cdot \text{s}]$ and $[\Omega] = [\text{V}/\text{A}]$, we obtain

$$\mu_{\text{Ag}} = 66,2 \frac{\text{cm}^2}{\text{Vs}}$$

$$\mu_{\text{Cu}} = 43,6 \frac{\text{cm}^2}{\text{Vs}}$$

$$\mu_{\text{Pb}} = 9,1 \frac{\text{cm}^2}{\text{Vs}}$$

Second Task: Derive numbers for the drift velocity v_D by considering a reasonable field strength.

- The mobility μ was defined as

$$\mu = \frac{v_D}{E}$$

or

$$v_D = \mu \cdot E$$

So what is a reasonable field strength in a metal?

- Easy. Consider a cube with side length $l = 1 \text{ cm}$. Its resistance R is given by

$$R = \frac{\rho \cdot l}{F} = \rho \Omega$$

- A **Cu** or **Ag** cube thus would have a resistance of about $1,5 \cdot 10^{-6} \Omega$. Applying a voltage of **1 V**, or equivalently a field strength of **1 V/cm** thus produces a current of $I = UI/R \approx 650\,000 \text{ A}$ or a current density $j = 650\,000 \text{ A/cm}^2$
- That seems to be an awfully large current. Yes, but it is the kind of current *density* encountered in integrated circuits! Think about it!
- Nevertheless, the wires in your house carry at most about **30 A** (above that the fuse blows) with a cross section of about **1 mm²**; so a reasonable current density is **3000 A/cm²**, which we will get for about $U = 1,5 \cdot 10^{-6} \Omega \cdot 3000 \text{ A} = 4,5 \text{ mV}$.
- For a rough estimate we then take a field strength of **5 mV/cm** and a mobility of **50 cm²/Vs** and obtain

$$v_D = 50 \cdot 5 \frac{\text{mV} \cdot \text{cm}^2}{\text{cm} \cdot \text{V} \cdot \text{s}} = 0,25 \frac{\text{cm}}{\text{s}} = 2,5 \frac{\text{mm}}{\text{s}}$$

That should come as some surprise! The electrons only have to move *very slowly on average* in the current direction (or rather, due to sign conventions, against it).

- Is that true, or did we make a mistake?
- It *is* true! However, it does *not* mean, that electrons will not run around like crazy inside the crystal, at very high speeds. It only means that their *net* movement in current anti-direction is very slow.
- Think of an single fly in a fly swarm. Even better [read the module](#) that discusses this analogy in detail. The flies are flying around at high speed like crazy - but the fly swarm is not going anywhere as long as it stays in place. There is then no drift velocity and no net fly current!