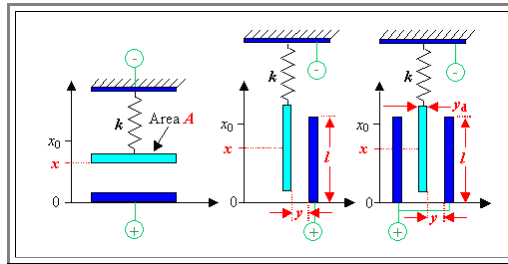


## Solution to Exercise 7.1-2

### Forces in Capacitive Structures

Here is the drawing once more to avoid jumping back and forth:



1. Find the proper relations for the forces pulling at the moveable plates for all three configurations.

The total energy  $E$  for all three configurations is simply given by

$$E = \int U \cdot I \cdot dt + \int k \cdot (x - x_0) \cdot dx$$

With the simple relations  $C = Q/U$ ,  $I = dQ/dt$  and therefore  $U \cdot I \cdot dt = (Q/C) \cdot (dQ/dt) \cdot dt = Q/C \cdot dQ$ , we obtain

$$E = \frac{Q^2}{2C} + \frac{k \cdot (x - x_0)^2}{2}$$

$$= \frac{C \cdot U^2}{2} + \frac{k \cdot (x - x_0)^2}{2}$$

$$F = -\frac{U^2}{2} \cdot \frac{dC}{dx} - k \cdot (x - x_0)$$

Without integration limits we cannot get proper signs (one energy term must decrease if the other one increases because we have energy conservation) - but that is not important here since we know that the spring force and the capacitive force must have opposite signs, and we are only interested in the capacitive force  $F_C$ .

For the capacity  $C$  and the force  $F_C = \frac{1}{2} U^2 \cdot (dC/dx)$  we obtain:

1. Configuration:

$$C_1 = \frac{\epsilon_0 \cdot A}{x}$$

$$F_1 = \frac{U^2}{2} \cdot \frac{\epsilon_0 \cdot A}{x^2}$$

2. Configuration:

$x^*$  is the (easy to calculate) plate overlap for zero voltage. But since it disappears upon differentiation, we do not need to spell it

$$C_2 = \frac{\epsilon_0 \cdot (x^* - x) \cdot h}{y}$$

$$F_2 = \frac{U^2}{2} \cdot \frac{\epsilon_0 \cdot h}{y}$$

out.

- 3. Configuration:  
 $C_3$  is simply given by 2  
 $C_2$  in parallel

$$C_3 = 2C_2 = \frac{2\epsilon_0 \cdot (3l/2 - x) \cdot h}{y}$$

$$F_3 = 2F_2 = U^2 \cdot \frac{\epsilon_0 \cdot h}{y}$$

- 2. Compare the relative strength of the first and third configuration.

- If we simply take the relation  $F_1/F_3$  for equal distances between the plates (i.e.  $x = y$ ), we obtain

$$\frac{F_1}{F_3} = \frac{A}{y \cdot h}$$

- Considering that  $y \cdot h \ll A$  for typical structures, configuration 1 can transmit much more force than the other ones for about identical size.

- 3. Discuss the pro and cons of the two configurations for driving an actual actuator.

In configuration 1 the force decreases with the square of the distance between the plates; in the extreme case of zero distance the plates would stick together *forever* (in reality a fuse will blow).

- The design rule is obvious: Use with extreme care!

In configuration 3 the force is independent on the position, which makes the design reasy. However, the force is relatively small.

- The consequences are obvious too: This is the preferred configuration, but you need to employ many combs to achieve sufficient force.