

Misfit Dislocations in Heterojunctions

Advanced

Calculating the critical thickness of a layer with lattice constant a_1 on top of a substrate with lattice constant a_2 can become rather involved, if all components contributing to the elastic energy are taken into account.

- In particular, you may want to consider the anisotropy of the situation, the effect of a finite thickness of the top layer, the real geometry with respect to the dislocations (their line energy depends on this and that, and they may be split into partial dislocations).
- Then, after arriving at a formula, you may chose to make all kinds of approximations.
- In the backbone part of the script we had a simple formula (taken from a paper of the very well known scientist Sir Peter **Hirsch**) which you can find in the [link](#) (together with some comments):

$$d_{\text{crit}} = \frac{b}{8 \cdot \pi \cdot f \cdot (1 + \nu)} \cdot \ln \frac{e \cdot d_{\text{crit}}}{r_0}$$

- With b = **Burgers vector** of the dislocations; usually somewhat smaller than a lattice constant, f = misfit parameter = $(a_1 - a_2)/a_1$, ν = Poisson ration $\approx 0,4$, e = really e = base of natural logarithms r_0 = inner core radius of the dislocation; again in the order of lattice constant.

Lets look at some other approaches

- A formula taking into account most everything going back to J. W. **Matthews** and A.E. **Blakeslee** (1974) , who pioneered this field of research, is

$$d_{\text{crit}} = \frac{b \cdot (1 - \nu) \cdot \cos^2 \Theta}{8 \cdot \pi \cdot (1 + \nu) \cdot f \cdot \cos \lambda} \cdot \left(\ln \left(\frac{d_{\text{crit}}}{b} \right) + 1 \right)$$

- with Θ = angle between the dislocation line and its Burgers vector, λ = angle between the slip direction and that line in the interface plane that is normal to the line of intersection between the slip plane and the interface.
- For simple systems ($\Theta = 90^\circ$ and $\lambda = 0^\circ$), we have

$$d_{\text{crit}} = \frac{b}{8 \cdot \pi \cdot f \cdot (1 + \nu)} \cdot \left(\ln \left(\frac{d_{\text{crit}}}{r_0} \right) + 1 \right)$$

- And that is Sir Peters equation if you insert $\ln(e)$ for the 1 in the \ln term.

While Sir Peter used the simple approximation

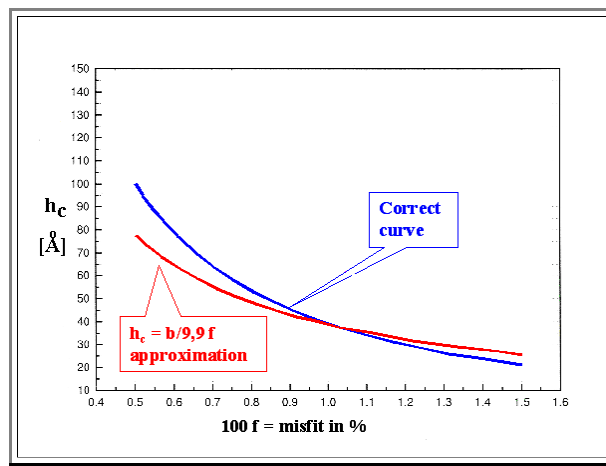
$$d_{\text{crit}} \approx \frac{b}{6f}$$

- the comparison with the (computer-generated) correct functional dependence suggests

$$d_{\text{crit}} \approx \frac{b}{9f}$$

- which is a bit different!

A plot of the full formula and the approximation looks like this:



Similar curves are contained in the books of [Mayer and Lau](#) or [Tu, Mayer and Feldmann](#); they supposedly use the same equation but show *rather different results*.

- Well, somewhere should be a mistake (maybe I made one?). In any case, it nicely demonstrates the point made in the backbone section: [Do not blindly believe a theory](#). In case of doubt, try it out.