

Solution to Exercise 2.1-2: Density of States for Lower Dimensions

Illustration

Calculate the density of states for a one-dimensional semiconductor ("quantum wire") and for the two-dimensional case.

Draw some conclusions from the results.

The number of states $Z(\mathbf{k})$ up to a wave vector \mathbf{k} is generally given by

$$Z(\mathbf{k}) = \frac{\text{Volume of "sphere" in } m \text{ dimensions}}{\text{Volume of state}}$$

The volume V_m of a "sphere" with radius k in m dimension is

$$V_m(k) = \begin{cases} \text{Volume of sphere} = 4/3 \pi \cdot k^3 & \text{for } m = 3 \\ \text{Volume} = \text{area of circle} = \pi \cdot k^2 & \text{for } m = 2 \\ \text{Volume} = \text{length} = 2k & \text{for } m = 1 \end{cases}$$

The density of states $D(E)$ follows by substituting the variable k by E via the dispersion relation $E(k)$ and by differentiation with respect to E .

One obtains the following relations:

$$D_m(E) \propto \begin{cases} (E)^{1/2} & \text{for } m = 3 \\ \text{const.} & \text{for } m = 2 \\ (E)^{-1/2} & \text{for } m = 1 \end{cases}$$

The consequences can be pretty dramatic. Consider, e.g. the concentration of electrons you can get in the three case for $E \approx 0 \text{ eV}$, i.e close to the band edge.

The question, of course, is: Are there 1-dim. and 2-dim. semiconductors? The answer is: yes – as soon as the other dimensions are small enough, we will encounter these cases. We will run across examples later in the lecture course.