

Accumulation

Advanced

This is the case where an electrical field of arbitrary origin *attracts the majority carriers*.

- Starting with the [Poisson equation for doped semiconductors and all dopants ionized](#), we [have seen](#) that we can approximate the situation by

$$\frac{d^2 \Delta E_C}{dx^2} = - e^2 \cdot N_D \epsilon \epsilon_0 \cdot \left(1 - \exp - \frac{\Delta E_C}{kT} \right)$$

- In contrast to the case of [quasi-neutrality](#), we now have (and the sign is important)

$$- \Delta E_C > kT$$

- This allows the approximation

$$1 - \exp - \frac{\Delta E_C}{kT} \approx - \exp - \frac{\Delta E_C}{kT}$$

- and the Poisson equation reduces to

$$\frac{d^2 \Delta E_C}{dx^2} = - \frac{e^2 \cdot N_D}{\epsilon \epsilon_0} \cdot \exp - \frac{\Delta E_C}{kT}$$

Using the [Debye length](#) $L_D = \{(\epsilon \epsilon_0 kT)/(e^2 N_D)\}^{1/2}$, or $N_D = \epsilon \epsilon_0 kT / e^2 L_D^2$, the *Poisson equation for accumulation* can be rewritten as

$$\frac{d^2 \Delta E_C}{dx^2} = - \frac{kT}{L_D^2} \cdot \exp - \frac{\Delta E_C}{kT}$$

- While this looks like a simple differential equation, it is not all that easy to solve it.

What we would need first, are defined *boundary conditions* so we can tackle the differential equation. There are no obvious candidates, so we have to think a little harder now.

- Accumulation means that we have some *surface charge* ρ that we put on the surface of the semiconductor (with our [fictive thin insulating](#) layer in between).
- We thus need to reformulate the differential equation so that *surface charge* can be included. the (not overly obvious) way to do this is to introduce the *electrical field strength* $E(x)$ as a new variable besides ΔE_C .
- For that we use the relation

$$\frac{d^2 \Delta E_C}{dx^2} = \frac{d}{dx} \left(\frac{d \Delta E_C}{dx} \right) = \frac{d}{dx} [e \cdot E(x)] = e \cdot \frac{dE(x)}{d \Delta E_C} \cdot \frac{d \Delta E_C}{dx} = e^2 \cdot E(x) \cdot \frac{dE(x)}{d \Delta E_C}$$

- We also made use of the equality $d \Delta E_C / d x = e \cdot E(x)$ with $E(x)$ = field strength.

Inserting and separating the variables (and omitting the "(x)" for clarity) gives

$$e^2 \cdot E \cdot dE = \frac{kT}{L_D^2} \cdot \exp - \frac{\Delta E_C}{kT} \cdot d \Delta E_C$$

- Tricky, but worth it. Now we can integrate both sides. The integrations run from far inside the bulk, i.e. from $E = 0$, to some value of E , and that means from $d\Delta E_C = 0$ to some corresponding value $d\Delta E_C$.

Omitting the integration (which is trivial), we obtain

$$\frac{E^2}{2} = \left(\frac{kT}{e \cdot L_D} \right)^2 \cdot \left(\exp - \frac{\Delta E_C}{kT} - 1 \right)$$

- i.e. an equation relating the amount of band bending at some position x to the electrical field strenght at this point, which is

$$E(x) = \pm \frac{kT}{e \cdot L_D} \left(2 \exp - \frac{\Delta E_C(x)}{kT} - 1 \right)^{1/2}$$

- For n-type semiconductors, which we are considering, ΔE_C is negative and large (i.e. $\Delta E_C \gg kT$) - and we may neglect the -1 , obtaining

$$E(x) \approx \pm \frac{kT}{e \cdot L_D} \cdot \left(2 \exp - \frac{\Delta E_C(x)}{kT} \right)^{1/2}$$

While this is fine, we still don't have the solution we want. We must now remember that there is a *simple relation tying surface charge to volume charge*.

- This is **Gauss law**, stating that the flux of the electrical field through a surface S is the integral over the components of E perpendicular to the surface.
- The charge is usually expressed in terms of charge density $\rho(x,y,z)$. Gauss law then states:

$$\iint_S \underline{E} \cdot \underline{n} \cdot d\mathbf{a} = \frac{1}{\epsilon \epsilon_0} \cdot \iiint_V \rho(x,y,z) \cdot dV$$

- With \underline{n} = normal vector of the surface S , $d\mathbf{a}$ = surface increment, dV = volume increment. For [more details](#) use the link.

For our case it means that we could replace the *total charge* ρ contained in a slice between $x = \infty$ (where there is no charge and the field strength is $E = E_{\text{bulk}} = 0$) and x , by a *surface* (or better areal) *charge* $\sigma_{\text{area}}(x)$ at x given by

$$\begin{aligned} \sigma_{\text{area}}(x) &= \epsilon \epsilon_0 \cdot (E(x) - E_{\text{bulk}}) = \epsilon \epsilon_0 \cdot E(x) \\ \sigma_{\text{area}}(x) &= \pm \frac{\epsilon \epsilon_0 \cdot kT}{e \cdot L_D} \cdot \left(2 \exp - \frac{\Delta E_C(x)}{kT} \right)^{1/2} \end{aligned}$$

The *total* amount of band-bending induced by a real *external surface charge* σ_{ex} is simply $\Delta E_C(x=0)$ which we call ΔE_C^0 :

$$\Delta E_C^0 = \pm 2kT \cdot \ln \frac{\sigma_{\text{ex}} \cdot e \cdot L_D}{2^{1/2} \cdot \epsilon \epsilon_0 \cdot kT}$$

- So we have all we need. The \pm sign came from the two solutions of the square root; we have to pick the correct one depending on the situation (holes or electrons considered).