

Solving the Poisson Equation for pn-Junctions

Advanced

■ We have the general equation for the space charge $\rho(x)$, and the Poisson equation:

$$\rho(x) = e \cdot \{n^h(x) - n^e(x) + N_D^+(x) - N_A^-(x)\}$$

$$\epsilon \epsilon_0 \cdot \frac{d^2 V(x)}{dx^2} = -\rho(x)$$

- $V(x)$ is the built-in potential resulting from the flow of majority carriers to the other side.

■ We consider a solution for the following conventions and approximations:

- The zero point of the electrostatic potential is identical to the valence band edge in the **p**-side of the junction shown in the [illustration](#).
- All dopants are ionized, i.e. $N_A = N_A^- = n^h$, and $N_D = N_D^+ = n^e$. This is always valid as long as the Fermi level is not very close to a band edge.

■ For the carrier density we have the general expression

$$n^{e,h} = N_{\text{eff}}^{e,h} \cdot \exp - \frac{\Delta E}{kT}$$

- and ΔE was $E_D - E_F$ for electrons and $E - E_A$ for holes.
- If $E_{D,A}$ is a function of x because the bands are bent (while E_F stays constant), we may write the energy difference as $\Delta E = \Delta E^0 + e \cdot V(x)$ with $\Delta E = \Delta E^0$ referring to the situation without band bending.
- The carrier concentration then becomes

$$\begin{aligned} n &= N_{\text{eff}} \cdot \exp - \frac{\Delta E + eV(x)}{kT} = N_{\text{eff}} \cdot \exp - \frac{\Delta E}{kT} \cdot \exp - \frac{eV(x)}{kT} \\ &= N_{A,D} \cdot \exp - \frac{e \cdot V(x)}{kT} \end{aligned}$$

- because the first term gives the concentration for $V(x) = 0$ and that is the dopant concentration in our approximation.
- We thus have for the carrier concentrations in equilibrium anywhere in the junction:

$$\begin{aligned} n^h(x) &= N_A \cdot \exp - \frac{e \cdot V(x)}{kT} \\ n^e(x) &= N_D \cdot \exp - \frac{e \cdot \{V(x) - V(n)\}}{kT} \end{aligned}$$

■ As soon as $V(x)$ deviates noticeably from its constant value of 0 or $V(n)$ - in other words: inside the space charge region - the carrier concentrations decrease exponentially from their values N_A or N_D far outside of the **SCR**. We therefore approximate their concentration by

$n^h = N_A$	for	$x < -d_A$
$n^h = 0$	for	$x > -d_A$
$n^e = N_D$	for	$x > d$
$n^e = 0$	for	$x < d$

With d_A, d_D = boundaries of the space charge region with $x = 0$ at the geometrical junction

The space charge then is *only* given by the concentration of the dopants. That's where we could have started right away, just plugging in the usual assumptions. We have

$\rho = N_A$	for	$-d_A < x < 0$
$\rho = N_D$	for	$0 < x < d_N$
$\rho = 0$	for	everywhere else

The Poisson equation then becomes

$\frac{d^2V}{dx^2} = 0$	for	$-\infty < x < -d_A$
$\frac{d^2V}{dx^2} = + \frac{e}{\epsilon\epsilon_0} N_A$	for	$-d_A < x < 0$
$\frac{d^2V}{dx^2} = - \frac{e}{\epsilon\epsilon_0} N_D$	for	$0 < x < d_D$
$\frac{d^2V}{dx^2} = 0$	for	$d_D < x < \infty$

In addition we have the boundary conditions:

$V = 0$	}	for	$x = -d_A$
$\frac{dV}{dx} = 0$			
$V = V(N)$	}	for	$x = d_D$
$\frac{dV}{dx} = 0$			
$d_A \cdot N_A = d_D \cdot N_D$	Charge neutrality		

The solutions are easily obtained, they are

$$V_A(x) = \frac{e}{2\epsilon\epsilon_0} \cdot N_A \cdot (d_A + x)^2 \quad \text{for} \quad -d_A < x < 0$$

$$V_D(x) = V(n) - \frac{e}{2\epsilon\epsilon_0} \cdot N_D \cdot (d_D - x)^2 \quad \text{for} \quad 0 < x < d_D$$

$$V(n) = \frac{e}{2\epsilon\epsilon_0} (N_A \cdot d_A^2 + N_D \cdot d_D^2)$$

The last equation comes from the condition of continuity at $x = 0$, i.e. $V_D(x = 0) = V_A(x = 0)$.

The two limits of the space charge region, d_A and d_D , as well as the field strength $E = -dV/dx$ in the SCR thus could be calculated if we would know $V(n)$.

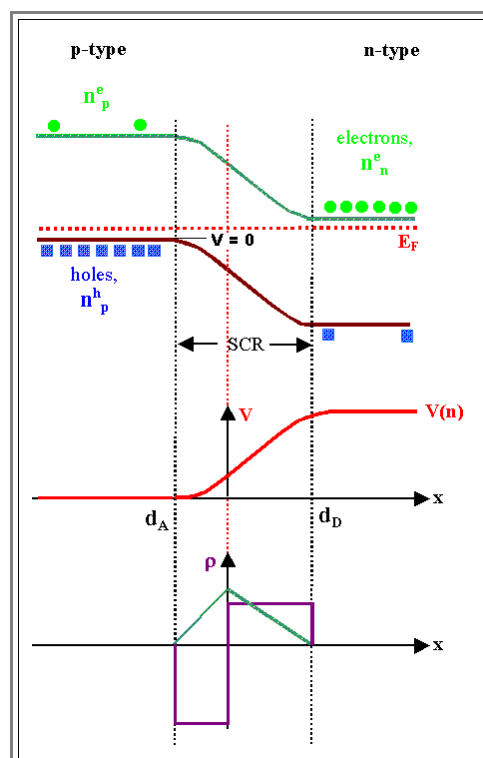
$V(n)$, of course, is the difference of the potential across the SCR and thus identical to $1/e$ times the difference of the Fermi energies before contact in thermal equilibrium, we have

$$V(n) = - \frac{E_{n_F} - E_{p_F}}{e}$$

If we superimpose an external voltage U , $V(n)$ becomes (watch out for the correct sign!).

$$V(n) = \frac{E_{n_F} - E_{p_F}}{e} \pm eU$$

The following illustration shows the whole situation in one drawing.



▶ We now can express the width d_{SCR} of the space charge region as

$$d_{\text{SCR}} = d_A + d_B = \frac{1}{e} \left(2 \epsilon_0 \cdot [\Delta E_F + e \cdot U_{\text{ex}}] \cdot \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \right)^{1/2}$$

● ΔE_F refers to the the difference of the Fermi energies before the contact and U_{ex} is the external voltage.