

Solution to Exercise 2.1-1

Derive and discuss numbers for μ and v_D

First Task: Derive numbers for the mobility μ .

- First we need typical conductivities and electron densities in *metals*, which we can take from the [table in the link](#).
- At the same time we expand the table a bit

Material	ρ [Ω cm]	σ [Ω^{-1} cm $^{-1}$]	Density d [10^3 kg m $^{-3}$]	Atomic weight w [1u = $1.66 \cdot 10^{-27}$ kg]	$n = d/w$ [m $^{-3}$]
Silver (Ag)	$1.6 \cdot 10^{-6}$	$6.2 \cdot 10^5$	10.49	107.9	$5.85 \cdot 10^{28}$
Copper (Cu)	$1.7 \cdot 10^{-6}$	$5.9 \cdot 10^5$	8.92	63.5	$8.46 \cdot 10^{28}$
Lead (Pb)	$21 \cdot 10^{-6}$	$4.8 \cdot 10^4$	11.34	207.2	$3.3 \cdot 10^{28}$

For the mobility μ we have [the equation](#)

$$\mu = \frac{\sigma}{q \cdot n}$$

With q = elementary charge = $1.60 \cdot 10^{-19}$ C and the density of electrons = density of atoms n calculated above, we obtain, for example for μ_{Ag}

$$\mu_{Ag} = \frac{6.2 \cdot 10^5}{1.6 \cdot 10^{-19} \cdot 5.85 \cdot 10^{28}} \frac{\text{m}^3}{\text{C} \cdot \Omega \cdot \text{cm}} = 66.2 \frac{\text{cm}^2}{\text{C} \cdot \Omega}$$

The unit is a bit strange, but remembering that $1 \text{ C} = 1 \text{ A} \cdot 1 \text{ s}$ and $1 \Omega = 1 \text{ V} / 1 \text{ A}$, we obtain

$$\mu_{Ag} = 66.2 \frac{\text{cm}^2}{\text{Vs}}$$

$$\mu_{Cu} = 43.6 \frac{\text{cm}^2}{\text{Vs}}$$

$$\mu_{Pb} = 9.1 \frac{\text{cm}^2}{\text{Vs}}$$

Second Task: Derive numbers for the drift velocity v_D by considering a reasonable field strength.

- The mobility μ was defined as

$$\mu = \frac{v_D}{E}$$

or

$$v_D = \mu \cdot E$$

So what is a reasonable field strength in a metal?

- Easy. Consider a cube with side length $l = 1 \text{ cm}$. Then, the relevant area is $F = 1 \text{ cm}^2$, and its resistance R is given by

$$R = \frac{\rho \cdot l}{F} = \rho [\Omega]$$

- A **Cu** or **Ag** cube thus would have a resistance of about $1.5 \cdot 10^{-6} \Omega$. Applying a voltage of **1 V**, or equivalently a field strength of **1 V/cm** thus produces a current of $I = U/R \approx 650\,000 \text{ A}$ or a current density of $j = 650\,000 \text{ A/cm}^2$
- That seems to be an awfully large current. Yes, but it is the kind of current *density* encountered in integrated circuits! Think about it!
- Nevertheless, the wires in your house carry at most about **30 A** (above that the fuse blows) with a cross section of about **1 mm²**; so a reasonable current density is **3000 A/cm²**, which we will get for about $U = 1.5 \cdot 10^{-6} \Omega \cdot 3000 \text{ A} = 4.5 \text{ mV}$.
- For a rough estimate we then take a field strength of **5 mV/cm** and a mobility of **50 cm²/Vs** and obtain

$$v_D = \frac{50 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \cdot 5 \frac{\text{mV}}{\text{cm}}}{1} = \frac{0.25 \frac{\text{cm}}{\text{s}}}{2.5} = \frac{\text{mm}}{\text{s}}$$

That should come as some surprise! The electrons only have to move *very slowly on average* in the current direction (or rather, due to sign conventions, against it).

- Is that true, or did we make a mistake?
- It *is* true! However, it does *not* mean that electrons will not run around like crazy inside the crystal, at very high speeds. It only means that their *net* movement in current (anti-)direction is very slow.
- Think of an single fly in a fly swarm (even better, [read the module](#) that discusses this analogy in detail). The flies are flying around at high speed like crazy – but the fly swarm is not going anywhere as long as it stays in place. There is then no drift velocity and no net fly current!