

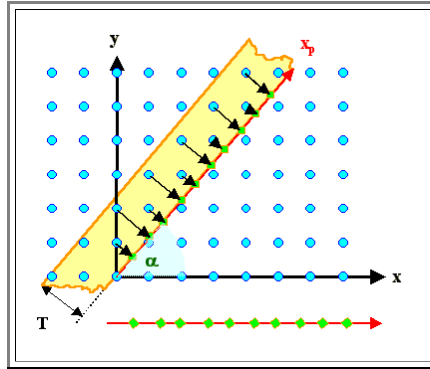
Quasikristalle II

This module will be in English because it is also used for other Hyperscripts.

Advanced

One of the more exciting (or frightening, depending on one's perspective to science) developments in understanding quasicrystals was the insight that quasicrystals can be constructed by projecting a *perfect* lattice in *6-dimensional space* onto a properly chosen *3-dimensional* subspace (which is the kind of space you and I know).

The gist of this outrageous statement is actually far easier to understand than it appears at first. For that we look at a simple analogue: We project a *perfect 2-dimensional lattice* on a *1-dimensional subspace*. How this is done is shown in the figure below.



Starting from a simple *2-dimensional* cubic lattice, we draw a straight line x_p at an angle α to the x -direction. The decisive point is that $\tan \alpha$ must be an *irrational* number, e.g. $(5)^{1/2}$ or $\pi/3$ or whatever. This makes sure that x_p will *not* touch another lattice point *ever*.

We then define an area (yellow) by drawing a line parallel to x_p at some distance T , which can be an arbitrary number.

Some lattice points will now be found within the yellow area, we project their position onto x_p .

The sequence of points obtained that way (shown at the bottom as the sequence of green "diamonds") is - by definition if you think about it - aperiodic; it will *never repeat*.

But it is not completely random either. There are only two different distances between points, their sequence just does not follow a periodic pattern.

We have actually produced a *one-dimensional quasicrystal*.

Now let's take a *six-dimensional space* and construct a cubic primitive lattice. No mathematician has the slightest problem doing that - you simply get a "hypercube" elementary cell with **64** corners and so on.

Now let's take a regular three-dimensional space. We make sure that the three-dimensional space is oriented relative to the six-dimensional space in such a way, that the six base vectors of the hypercube are projected onto the three-dimensional space with the fivefold symmetry of an *icosahedra*. (An icosahedra is one of those regular "eders" with triangular faces, where always five triangles sort of group around an axis with five-fold symmetry.)

Interestingly, instead of some $\tan \alpha$ the number $N = [1 + (5)^{1/2}]$ appears - which is the "magic number of the **golden ratio**". This may or may not mean something special.

Now you define some neighborhood around your three-dimensional space and start projecting - you will get the exact arrangement of atoms in a real three-dimensional quasicrystal.

Don't worry if you can not imagine all this - nobody can.

Only worry if you can't see that there is a sophisticated, but nevertheless rather clear-cut mathematical procedure of how to construct a three-dimensional point sequence by all this hypercube projection stuff.

If you did not worry so far, you should now, pondering the question: *What does it mean?*

Again, who knows for sure? But we do know that there is some meaning. Let's just look at *one* example.

In three-dimensional quasicrystals we find some entities that look and behave exactly like *dislocations* (a one-dimensional defect in three-dimensional *lattices*). These whatever-they-are entities come into being after some deformation like *real* dislocations, they move through the quasicrystal like *real* dislocation, they look like *real* dislocations in the electron microscope - but they simply cannot be *real* dislocations because *real* dislocations can *only* exist in *periodic* lattices.

Turns out they are dislocations in the *six-dimensional periodic lattice* - no problem to define dislocations there. What we see is sort of what is left in the three-dimensional space where atoms live (not to mention you and me).

And while *real* dislocations are always characterized by their *Burgers vector* - just a regular vector with three components - the quasicrystal sort-of-dislocations need a vector with *six* components for their characterization.

● Mercifully enough, it turns out that the particulars of the projection scheme always allow to decompose the *one* six-dimensional Burgers vector into *two* three dimensional ones. These two regular vectors have a precise meaning as to what this dislocation-like thing does to the quasicrystal when it moves through it.

▮ All in all - quasicrystals, quite unexpectedly, link materials science with rather involved and rather unexpected math. It will be exciting to witness what will come from all this during the next **10** or **20** years.