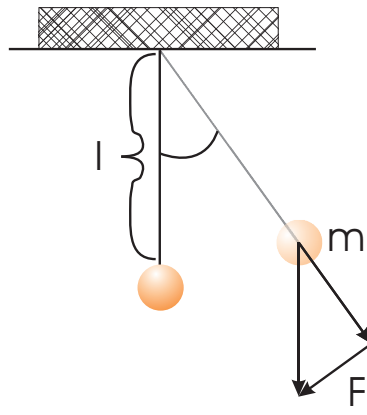


3.6.1 Example: Transformation of a non linear problem into a linear problem

Motion of a pendulum:



$$s = l\varphi \quad F_s = -mg \sin \varphi$$

Newton:

$$\begin{aligned} F_s = m\ddot{s} &\rightarrow m\ddot{s} = -mg \sin \varphi \\ l\ddot{\varphi} + g \sin \varphi = 0 &\rightarrow \ddot{\varphi} + \omega^2 \sin \varphi = 0 \quad (\star) \\ &\varphi(t) = ? \text{ function is looked for ?} \end{aligned}$$

Eq. $(\star)$ is extremely complicated since it is non-linear because of the $\sin \varphi$. For small φ : $\sin \varphi \approx \varphi$ $|\sin \varphi - \varphi| \leq \frac{\varphi^3}{3!} \approx 10^{-3}$ for $\varphi \leq 10^\circ$

$$\rightarrow \ddot{f}(t) + \omega^2 f(t) = 0 \quad \frac{d^2 \varphi}{dt^2} + \omega^2 \varphi(t) = 0 \leftarrow \text{linear equation}$$

See also exercises:

$$f(t) = A_1 e^{i\omega t} + A_2 e^{-i\omega t} = \varphi \cos(\omega t) + \frac{\varphi}{\omega} \sin(\omega t)$$