

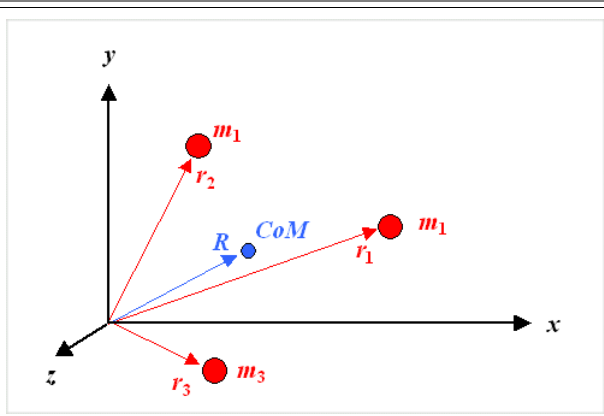
## Center of Mass

Quite often the term "**center of gravity**" is used as a synonym for center of mass. That is correct as long as the gravity force is constant everywhere. This is the case in an extremely good approximation on the surface of this planet, so we won't worry about it and consider both centers to be the same. We may then consider some body on earth that is at rest, i.e. not moving or rotating - despite the force of gravity acting on it! That requires that some supported at least at one point.

- Well, for starters we note that the sum of all forces needs to be zero if we do not want the body to move. That is our *first* condition.  
We know that on each mass element  $m_i$  of the body acts the gravity force  $m_i \cdot g$ . The total force then is  $F_g = \sum m_i \cdot g = Mg$  with  $M$ =total mass of the body.  
The trivial result is that whatever holds up the body (e.g. the pencil end holding up the thega in [this picture](#)) must feels (or exert) this force *upwards*, compensating gravity's *downwards* pull so the total force is zero.
- The proper question to ask now is: "If I want to support the body on just one point, where does that point have to be?"  
From the math we don't know anything about the location of this crucial point of support yet. But we know, of course, that this point needs to be right under the center of mass. Can we deduce that? The force equilibrium done above won't be enough..

We need to consider the *second* condition. If a body supported at one point is supposed to be at rest. It mustn't move but it mustn't *rotate* either! This demands that the sum of all *torques* must be zero, too!

- If we consider our body to consist of a collection of  $N$  individual masses  $m_i$ , sitting at the end of vectors  $r_i$  defined in some coordinate system (see below), we have



$$\sum_{i=1}^N m_i (r_i - R) = 0 \quad \text{or} \quad R = \frac{1}{M} \sum_{i=1}^N m_i \cdot r_i$$

- $R$  is the vector to the center of mass, and  $M$  is the total mass of all the  $N$  "mass points". The expression  $m_i (r_i - R)$  gives the torque acting on the mass  $m_i$  with respect to the center of mass, which is the only point around which a rotation can take place. Do you see why? No? Well, if the body would rotate around any other point, its center of mass would move, something we excluded by definition.

If we do not have a succession of individual mass points but some body  $B$  with a density  $\rho(r)$ , we move from a summation to an integration and obtain

$$\int \int \int_B \rho(r) \cdot (r - R) \cdot dV = 0 \quad \text{or} \quad R = \frac{1}{M} \int \int \int_B \rho(r) \cdot r \cdot dV$$

- That's all. Of course, the integration over a complex body made from materials with different densities (like a sword with a hilt) is not easy to do by hand. But we either do it by computer or simply find it experimentally. This is rather easy, even for [curved swords](#).