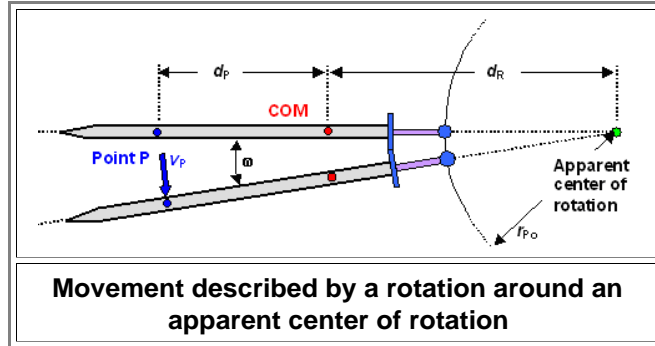


Angular velocity and Switching Systems

Illustration

This is an Illustration and not a science link because it just illustrates your thought processes. I'm sure that everything in here you could figure out yourself. Angela Merkel could (but possibly not D. Trump).

- We use the picture from the "[Hitting Something](#)" subchapter. A sword rotates around an apparent (or instantaneous) center of rotation. How fast does some point **P** somewhere on the blade move?



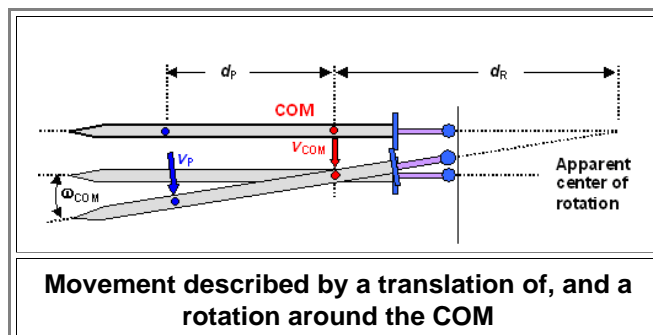
We assume of course a constant rotational speed or velocity. That means that the sword will finish a full circle in some time T_{cycle} . It thus makes $1/T_{\text{cycle}} = \nu$ full rotations per time unit. As a unit for the rotational speed you thus could give the number of revolutions *per minute* and call that "*rpm*". You just as well could do the number of revolutions *per second* and then you would call that the **cycle frequency** ν with the unit "hertz" [Hz]. And yes, **1 Hz = 1/s**.

- So how fast does a point **P** travel that is found at some distance r_p from the apparent center of rotation? Since it travels on a perfect circle with radius r_p , it covers a distance equal to the circumference C_p of a circle with radius r_p for every revolution. C_p is equal to $2\pi r_p$ as even D.T. might know. Since we have ν rounds per second, the total distance covered in 1 second and thus the speed is $2\pi\nu \cdot r_p$. That gives us an idea: We can save a lot of writing if we don't use the cycle frequency $\nu_{\text{cycle}} = 1/T_{\text{cycle}}$ but something called the **circular frequency** (or circle frequency) $\omega = 2\pi\nu$. It is as simple as that. Using ω instead of ν or $1/T$ or, God forbid, *rpm*, makes writing equations much more efficient. So everybody who is anybody uses the circular frequency and nothing else for rotational velocities.
- So how fast does a point **P** at some distance r_p from the apparent center of rotation travel? Here it is:

$$\begin{aligned} v_p &= \omega \cdot r_p \\ &= \omega \cdot (d_p + d_r) \end{aligned}$$

for the point **P** shown

So far, so easy. But what happens if we do not describe the movement of the sword as a rotation around the apparent center of rotation (COM) but, in the way discussed at length in the [backbone](#), as a combination of a pure translation for the center of mass plus a pure rotation around the center of mass. This is schematically shown below.



- The point P then moves with the speed of the COM which is $\mathbf{v}_{\text{COM}} = \omega \cdot \mathbf{d}_r$ plus the rotational speed for the rotation around the COM which is $\mathbf{v}_{\text{rot, COM}} = \omega_{\text{COM}} \cdot \mathbf{d}_p$. The circle frequency ω_{COM} for the rotation around the center of mass is identical to the one we had before since, as the picture shows, the same rotation angle applies, i.e. $\omega_{\text{COM}} = \omega$,
This leaves us with

$$\begin{aligned} \mathbf{v}_P &= \omega_{\text{COM}} \cdot \mathbf{d}_p + \omega \cdot \mathbf{d}_r \\ &= \omega \cdot (\mathbf{d}_p + \mathbf{d}_r) \end{aligned}$$

- Same as above. It also works for the kinetic energies E_{kin} . They must be the same in both cases. With I_A = moment of inertia for rotation around the apparent center of rotation, and I_{COM} = moment of inertia for rotation around the COM, we get for the kinetic energy using the rotation around the apparent center of rotations:

$$\begin{aligned} E_{\text{kin}} &= \frac{1}{2} I_A \cdot \omega^2 \\ &= \frac{1}{2} (I_{\text{COM}} + m d_R^2) \omega^2 \end{aligned}$$

- We used the [parallel axis theorem](#) for this; m is the mass of the sword. Calculating the kinetic energy for the COM system we get

$$\begin{aligned} E_{\text{kin}} &= \frac{1}{2} (I_{\text{COM}} \omega^2 + m v_{\text{COM}}^2) \\ &= \frac{1}{2} (I_{\text{COM}} + m d_R^2) \omega^2 \end{aligned}$$

Just as it should be. I rest my case.