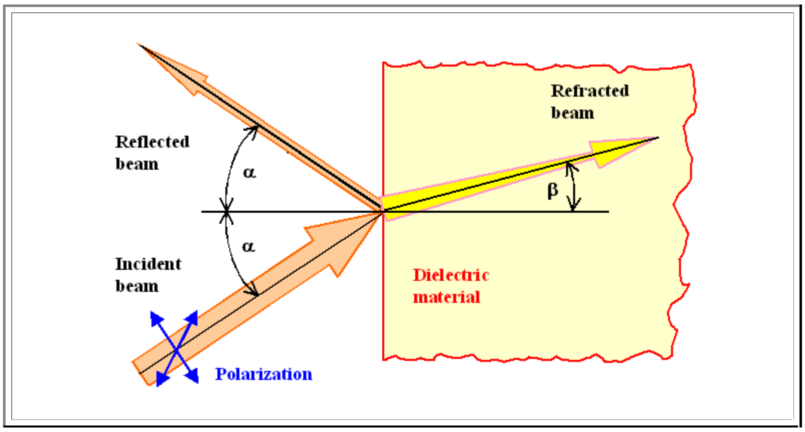
3.7 Dielectrics and Optics

3.7.1 Basics

- This subchapter can easily be turned into a whole lecture course, so it is impossible to derive all the interesting relations *and* to cover anything in depth. This subchapter therefore just tries to give a strong flavor of the topic.
- We know, of course, that the **index of refraction *n*** of a non-magnetic material is linked to the dielectric constant ϵ_r via a simple relation, which is a rather direct result of the Maxwell equations.

$$n = (\epsilon_r)^{1/2}$$

- But in learning about the origin of the dielectric constant, we have progressed from a simple constant ϵ_r to a complex **dielectric function** with frequency dependent real and imaginary parts.
  - What happens to ***n*** then? How do we transfer the wealth of additional information contained in the *dielectric function* to optical properties, which are to a large degreee encoded in the *index of refraction*?
- Well, you probably guessed it: We switch to a **complex index of refraction**!
  - But before we do this, let's ask ourselves what we actually want to find out. What are the optical properties that we like to know and that are *not* contained in a simple index of refraction?
  - Lets look at the *paradigmatic* experiment in optics and see what we *should* know, what we *already* know, and what we *do not* yet know.



- What we have is an electromagnetic wave, an *incident* beam (traveling in vacuum to keep things easy), which impinges on our dielectric material. As a result we obtain a *reflected* beam traveling in vacuum and a *refracted* beam which travels through the material. What do we know about the three beams?
  - The *incident beam* is characterized by its wavelength λ_i, its frequency ν_i and its velocity c_0, the direction of its **polarization** in some coordinate system of our choice, and the arbitrary angle of incidence α. We know, it is hoped, the simple **dispersion relation** for vacuum.

$$c_0 = \nu_i \cdot \lambda_i$$

  - c_0 is, of course, the velocity of light in vacuum, an absoute constant of nature.
- The *incident beam* also has a certain *amplitude* of the electric field (and of the magnetic field, of course) which we call E_0. The *intensity* I_i of the light that the incident beams embodies, i.e. the energy flow, is proportional to E_0^2 - never mix up the two!
- The *reflected beam* follows one of the basic laws of optics, i.e. *angle of incidence* = ***angle of emergence***, and its wavelength, frequency and magnitude of velocity are identical to that of the incident beam.
  - What we do *not know* is its *amplitude* and its *polarization*, and these two quantities must somehow depend on the properties of the incident beam *and* the properties of the dielectric.
- If we now consider the *refracted beam*, we know that it travels under an angle β, has the same frequency as the incident beam, but a wavelength λ_d and a velocity c that is different from λ_i and c_0.
  - Moreover, we must expect that it is *damped* or *attenuated* , i.e. that its amplitude decreases as a function of penetration depth (this is indicated by decreasing thickness of the arrow above). All parameters of the refracted beam may depend on the polarization of the incident beam.
  - Again, *basic* optics teaches that there are some simple relations. We have

$\dfrac{\sin \alpha}{\sin \beta} = n$	**Snellius law**
$n = \dfrac{c_0}{c}$	**From Maxwell equations**
$c = \nu_i \cdot \lambda_d$	**Always valid**
$\lambda_d = \dfrac{1}{n} \cdot \lambda_i$	**From the equations above**

- A bit more involved is another basic relations coming from the Maxwell equations. It is the equation linking ***c***, the speed of light in a material to the material "constants" ϵ_r and the corresponding magnetic permeability μ_0 of vacuum and μ_r of the material via

$$c = \frac{1}{(\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_r)^{1/2}}$$

- Since most optical materials are not magnetic, i.e. $\mu_r = 1$, we obtain for the index of refraction of a dielectric material our relation from above.

$$n = \frac{c_0}{c} = \frac{(\mu_0 \cdot \mu_r \cdot \epsilon_0 \cdot \epsilon_r)^{1/2}}{(\mu_0 \cdot \epsilon_0)^{1/2}} = \epsilon_r^{1/2}$$

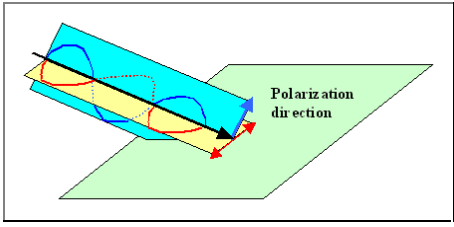
- Consult the basic optics module if you have problems so far.

If we now look at not-so-basic optics, we encounter the **Fresnel laws** of diffraction.

- Essentially, the *Fresnel laws* give the *intensity of the reflected beam* as a function of the *angle of incidence* , the *polarization* of the incident beam, and the *index of refraction* of the material.
- The Fresnel laws are not particularly easy to obtain (consult the basic module Fresnel laws), but the results are easy.

First, we must distinguish between the two basic polarization cases possible:

- The incident light might be polarized in such a way that the vector of the electrical field **E** lies either *in* the plane of the material, or *perpendicular* to it, as shown below. Anything in between than can be decomposed into the two basic cases.



- Lets call the amplitudes of the reflected beam A_{para} for the case of the polarization being *parallel* to the plane (= surface of the dielectric), and A_{perp} for the case of the polarization being *perpendicular* to the plane (blue case) as shown above. For a unit amplitude of the incident beam, the Fresnel laws then state

$$A_{para} = -\frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)} \qquad A_{perp} = -\frac{\tan(\alpha - \beta)}{\tan(\alpha + \beta)}$$

- We can substitute the angle β by using the relation from above and the resulting equations then give the intensity of the reflected light as a function of the material parameter ***n*** . Possible, but the resulting equations are no longer simple.
  - In order to stay simple and focus on the essentials, we will now consider only cases with about *perpendicular* incidence, i.e. $\alpha \approx 0^o$. This makes everything much easier.
  - At *small* angles we may substitute the argument of the **sin** or **tan** for the full function, and obtain for *both* polarizations

$$A \approx -\frac{(\alpha - \beta)}{(\alpha + \beta)}$$

  - Using the expression for ***n*** from above for small angles too, we obtain

$$n = \frac{\sin\alpha}{\sin\beta} \approx \frac{\alpha}{\beta}$$

  - Now we keep in mind that we are usually interested in intensities , and not in amplitudes. Putting everything together, we obtain for the reflectivity ***R*** , defined as the ratio of the intensity I_r of the reflected beam to the intensity I_i of the incident beam for almost perpendicular incidence

$$R = \frac{I_r}{I_i} = \frac{(n-1)^2}{(n+1)^2}$$

- The grand total of all of this is that if we know ***n*** and some basics about optics, we can answer most, *but not all* of the questions from above. But so far we also did not use a *complex* index of refraction either.
  - In essence, what is missing is any statement about the *attenuation* of the refracted beam, the **damping of the light** inside the dielectric - it is simply not contained in the equations presented so far.
  - This cannot be right. Electromagnetic radiation does not penetrate arbitrarily thick (and still perfect) dielectrics - it gets pretty dark, for example, in deep water even if it is perfectly clear.
  - In not answering the "damping" question, we even raise a new question: If we include damping in the consideration of wave propagation inside a dielectric, does it change the simple equations given above?
- The *bad* news is: It does! But relax: The *good* news is:
  - All we have to do is to exchange the "simple" refractive index ***n*** by a *complex refractive index* ***n**** that is directly tied to the complex dielectric function, and everything is taken care of.
  - We will see how this works in the next paragraph.