Frequency Dependence of Orientation Polarization

Advanced

- How do we get to the time- and frequency dependence ***P*(t)** and ***P*(ω)**, respectively, of the orientation polarization without "cutting corners" as in the backbone?
  - While in principle each function is just the Fourier transform of the other, it is not so easy to actually do the required math. It is probably best, to start with the differential equation that describes the system.
- Within the "relaxation time approximation" always used for those cases we have

$$\frac{dP(t)}{dt} = -\frac{P}{\tau} + S(t)$$

  - ***S*(*t*)** is some disturbance or signal or input - whichever term you prefer - that has some time dependence. We need it because otherwise the system would be "dead" and not doing anything after at most one decay if we pick suitable starting conditions.
- Whatever happens, we can always write ***P*(*t*)** and **S(*t*)** as a Fourier series, or, more general, as Fourier integral of the correlated ***P*(ω)** and ***S*(ω)** "spectra" of the time functions. Doing this we have

$$P(t) = \int_{-\infty}^{\infty} P(\omega) \cdot \exp(i\omega t) \cdot dt$$

$$S(t) = \int_{-\infty}^{\infty} S(\omega) \cdot \exp(i\omega t) \cdot dt$$

  - ***P*(ω) · exp(iω*t*)** and ***S*(ω) · exp(iω*t*)** are the Fourier "coefficients" (with values for every ω, not just harmonics as in Fourier series) for the time functions.
- We now have a linear differential equation that is solved by some ***P*(*t*)** which can be expressed as a Fourier transform and that implies that all Fourier coefficients (and any superposition thereof) also solve the differential equation. Inserting the Fourier coefficients directly gives

$$\frac{d\{P(\omega) \cdot \exp(i\omega t)\}}{dt} = i\omega \cdot P(\omega)\exp(i\omega t) = -\frac{P(\omega) \cdot \exp(i\omega t)}{\tau} + S(\omega) \cdot \exp(i\omega t)$$

  - From this we obtain

$$P(\omega) \cdot (\,i\omega + 1/\tau) = S(\omega)$$

- If we define ω_0 **= 1/τ** (or **= A/τ** if we want to be more general) we now have a simple relation between the Fourier components of input ***S*** and output ***P***:

$$P(\omega) = \frac{1}{\omega_0 - i\omega} \cdot S(\omega)$$

- The dielectric function **ϵ(ω)** that we are trying to calculate is simple the relation between the output ***P*(ω)** and the input ***S*(ω)**, what we get is

$$\epsilon(\omega) := \frac{P(\omega)}{S(\omega)} = \frac{1}{\omega_0 - i\omega}$$

- Considering that the disturbance ***S*(**ω**)** must have the dimension of an electrical field, forces us to conclude that it actually must be an electrical field, and we could just as well write ***E*(**ω**)**. What we have then is essentially the dielectric function as discussed in the backbone.